Electronic Journal of University of Aden for Basic and Applied Sciences
EJUA-BA Vol. 1 No. 1 (2020)
https://ejua.net/index.php/EJUA-BA/article/view/10
ISSN: 2708-0684

RESEARCH ARTICLE

TRANSFORMATION AND SUMMATION FORMULAS FOR EXTON'S FUNCTIONS K_{11}, K_{14} AND K_{15}

Ahmed Ali Atash[1,*] and Hussein Saleh Bellehaj[2]

[1,2]*Department of Mathematics, Faculty of Education - Shabwah, University of Aden, Aden, Yemen*

***Corresponding author: Ahmed Ali Atash E-mail: ah-a-atash@hotmail.com**

Received: 31 Jan 2020 / Accepted: 18 Feb 2020 / Published online: 07 Mar 2020

Abstract

The aim of the present paper is to establish some transformation formulas for Exton's quadruple hypergeometric functions K_{11}, K_{14} and K_{15}. Several summation formulas for K_{11}, K_{14} and K_{15} are also derived as an applications of our main results with the help of classical summation theorems.

Keywords: Transformation formulas, Summation formulas, Hypergeometric functions, Classical summation theorems.

2010 Mathematics Subject Classification - 33B15, 33C05, 33C20, 33C65.

1. Introduction

The generalized hypergeometric function pFq with p numerator parameters and q denominator parameters is defined by (see **[7, p.42]**):

$$ {}_pF_q\left[\begin{matrix} a_1, \ldots, a_p ; \\ b_1, \ldots, b_q ; \end{matrix} z\right] = \sum_{n=0}^{\infty} \frac{(a_1)_n \cdots (a_p)_n}{(b_1)_n \cdots (b_q)_n} \frac{z^n}{n!}, \qquad (1.1)$$

where $(\lambda)_n$ is the Pochhammer's symbol defined by (see **[7, p. 21]**)

$$(\lambda)_n = \frac{\Gamma(\lambda+n)}{\Gamma(\lambda)} = \begin{cases} 1 & (n=0) \\ \lambda(\lambda+1)(\lambda+2)....(\lambda+n-1) & (n \in N) \end{cases} \qquad (1.2)$$

and $\Gamma(\lambda)$ is the Gamma function defined by

$$\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t}\, dt \quad , \quad \mathrm{Re}(z) > 0 \ . \qquad (1.3)$$

In the theory of hypergeometric and generalized hypergeometric series, classical summation theorems such as those of Gauss, Gauss's second, Bailey, Kummer and Dixon play an important role. Applications of the above-mentioned theorem are now well known (see, **[1], [4], [6]**). For the purposes of our present work, we require the following classical summation theorems:

Gauss's theorem (see, **[5, p.49]**)

$$ {}_2F_1\left[\begin{matrix} a, b; \\ c \ ; \end{matrix} 1\right] = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}, \quad R(c-a-b) > 0 \qquad (1.4)$$

Gauss's second theorem (see**[5, p.69]**)

$$ {}_2F_1\left[\begin{matrix} a, b & ; \frac{1}{2} \\ \frac{1}{2}(a+b+1) & ; 2 \end{matrix}\right] = \frac{\Gamma(\frac{1}{2})\ \Gamma(\frac{1}{2}a+\frac{1}{2}b+\frac{1}{2})}{\Gamma(\frac{1}{2}a+\frac{1}{2})\Gamma(\frac{1}{2}b+\frac{1}{2})} \qquad (1.5)$$

Bailey's theorem (see **[5, p.69]**)

$$ {}_2F_1\left[\begin{matrix} a, 1-a; & \frac{1}{2} \\ c & ; 2 \end{matrix}\right] = \frac{\Gamma(\frac{1}{2}c)\ \Gamma(\frac{1}{2}c+\frac{1}{2})}{\Gamma(\frac{1}{2}c+\frac{1}{2}a)\Gamma(\frac{1}{2}c-\frac{1}{2}a+\frac{1}{2})} \qquad (1.6)$$

Kummer's theorem (see **[5, p.68]**)

$$ {}_2F_1\left[\begin{matrix} a, b & ; \\ 1+a-b & ; \end{matrix} -1\right] = \frac{\Gamma(1+a-b)\Gamma(\frac{1}{2})}{2^a\Gamma(1+\frac{1}{2}a-b)\Gamma(\frac{1}{2}a+\frac{1}{2})} \qquad (1.7)$$

Dixon's theorem (see **[5, p.92]**)

$$ {}_3F_2\left[\begin{matrix} a, b, c & ; \\ 1+a-b, 1+a-c & ; \end{matrix} 1\right]$$

$$= \frac{\Gamma(1+\frac{1}{2}a)\ \Gamma(1+a-b)\ \Gamma(1+a-c)\ \Gamma(1+\frac{1}{2}a-b-c)}{\Gamma(1+a)\ \Gamma(1+\frac{1}{2}a-b)\Gamma(1+\frac{1}{2}a-c)\ \Gamma(1+a-b-c)} \qquad (1.8)$$

Lauricella's function $F_D^{(3)}$ (see, **[55, p.61]**)

$$F_D^{(3)}(a,b_1,b_2,b_3;c;1,1,1)=\frac{\Gamma(c)\,\Gamma(c-a-b_1-b_2-b_3)}{\Gamma(c-a)\Gamma(c-b_1-b_2-b_3)}, \quad (1.9)$$

where $F_D^{(3)}$ is Lauricella's function of three variables defined as follows (see **[7,p.60]**):

$$F_D^{(3)}(a,b_1,b_2,b_3;c;x,y,z)$$

$$=\sum_{m,\,n,\,p=0}^{\infty}\frac{(a)_{m+n+p}(b_1)_m(b_2)_n(b_3)_p}{(c)_{m+n+p}}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!} \quad (1.10)$$

$\max\{|x|,|y|,|z|\}<1$.

The Exton's quadruple hypergeometric functions K_{11}, K_{14} and K_{15} are defined by [2] (see also **[3, p.78-79]**) as follows:

$$K_{11}(a,a,a,a;b_1,b_2,b_3,b_4,c,c,c,d;x,y,z,t)$$

$$=\sum_{m,n,p,q=0}^{\infty}\frac{(a)_{m+n+p+q}(b_1)_m(b_2)_n(b_3)_p(b_4)_q\,x^m y^n z^p t^q}{(c)_{m+n+p}\,(d)_q\quad m!\,n!\,p!\,q!} \quad (1.11)$$

$$K_{14}(a,a,a,c_3;b,c_1,c_2,b;d,d,d,d;x,y,z,t)$$

$$=\sum_{m,n,p,q=0}^{\infty}\frac{(a)_{m+n+p}(c_3)_q(b)_{m+q}(c_1)_n(c_2)_p\,x^m y^n z^p t^q}{(d)_{m+n+p+q}\quad m!\,n!\,p!\,q!} \quad (1.12)$$

$$K_{15}(a,a,a,b_5;b_1,b_2,b_3,b_4;c,c,c,c;x,y,z,t)$$

$$=\sum_{m,n,p,q=0}^{\infty}\frac{(a)_{m+n+p}(b_5)_q(b_1)_m(b_2)_n(b_3)_p(b_4)_q\,x^m y^n z^p t^q}{(c)_{m+n+p+q}\quad m!\,n!\,p!\,q!}. \quad (1.13)$$

2. Main transformation formulas for K_{11}, K_{14} and K_{15}

In this section, we apply (1.9) to establish five transformation formulas for Exton's quadruple hypergeometric functions K_{11}, K_{14} and K_{15} as follows:

Theorem 2.1. The following transformation formula for K_{11} holds true:

$$K_{11}(a,a,a,a;b_1,b_2,b_3,b_4;c,c,c,d;1,1,1,t)$$

$$=\frac{\Gamma(c)\Gamma(c-a-b_1-b_2-b_3)}{\Gamma(c-a)\Gamma(c-b_1-b_2-b_3)}$$

$$\times\,{}_3F_2\left[\begin{matrix}a,b_4,1-c+a & ;\\ d,1-c+a+b_1+b_2+b_3; \end{matrix}\,t\right] \quad (2.1)$$

Proof. From the definition (1.11) of K_{11}, we have

$$K_{11}(a,a,a,a;b_1,b_2,b_3,b_4;c,c,c,d;1,1,1,t)$$

$$=\sum_{q=0}^{\infty}\frac{(a)_q(b_4)_q\;t^q}{(d)_q\;\;q!}F_D^{(3)}[a+q,b_1,b_2,b_3;c;1,1,1].$$

Now, using (1.9), we have

$$K_{11}(a,a,a,a;b_1,b_2,b_3,b_4;c,c,c,d;1,1,1,t)$$

$$=\sum_{q=0}^{\infty}\frac{(a)_q(b_4)_q\;t^q}{(d)_q\;\;q!}\frac{\Gamma(c)\Gamma(c-a-q-b_1-b_2-b_3)}{\Gamma(c-a-q)\Gamma(c-b_1-b_2-b_3)}$$

Next, using the following result (see **[p.22,7]**)

$$(a)_{-n}=\frac{(-1)^n}{(1-a)_n},\quad n=1,2,3,....\,;a\neq 0,\pm1,\pm2,....\quad ,$$

we have

$$K_{11}(a,a,a,a;b_1,b_2,b_3,b_4;c,c,c,d;1,1,1,t)$$

$$=\frac{\Gamma(c)\Gamma(c-a-b_1-b_2-b_3)}{\Gamma(c-a)\Gamma(c-b_1-b_2-b_3)}$$

$$\times\sum_{q=0}^{\infty}\frac{(a)_q(b_4)_q(1-c+a)_q\;\;t^q}{(d)_q(1-c+a+b_1+b_2+b_3)_q\;\;q!}$$

$$=\frac{\Gamma(c)\Gamma(c-a-b_1-b_2-b_3)}{\Gamma(c-a)\Gamma(c-b_1-b_2-b_3)}$$

$$\times{}_3F_2\left[\begin{matrix}a,b_4,1-c+a & ;\\ d,1-c+a+b_1+b_2+b_3; \end{matrix}\,t\right].$$

This completes the proof of (2.1).

Remark 2.1. On taking $b_4=1-c+a+b_1+b_2+b_3$ in (2.1), we obtain the following transformation formula:

Corollary 2.1. The following transformation formula for K_{11} holds true:

$$K_{11}(a,a,a,a;b_1,b_2,b_3,1+a-c+b_1+b_2+b_3;c,c,c,d;1,1,1,t)$$

$$=\frac{\Gamma(c)\Gamma(c-a-b_1-b_2-b_3)}{\Gamma(c-a)\Gamma(c-b_1-b_2-b_3)}\times{}_2F_1\left[\begin{matrix}a,1-c+a; \\ d\quad ;\end{matrix}\,t\right]. \quad (2.2)$$

Theorem 2.2. The following transformation formula for K_{14} holds true:

$$K_{14}(a,a,a,c_3;b,c_1,c_2,b;d,d,d,d;1,1,1,t)$$

$$=\frac{\Gamma(d)\Gamma(d-a-b-c_1-c_2)}{\Gamma(d-a)\Gamma(d-b-c_1-c_2)}\times {}_2F_1\left[\begin{matrix}b,c_3\,;\\ d-a\,;\end{matrix}t\right] \quad (2.3)$$

Proof. From the definition (1.12) of K_{14}, we have

$$K_{14}(a,a,a,c_3;b,c_1,c_2,b;d,d,d,d;1,1,1,t)$$

$$=\sum_{q=0}^{\infty}\frac{(b)_q(c_3)_q\ t^q}{(d)_q\ q!}F_D^{(3)}[a,b+q,c_1,c_2;d+q;1,1,1].$$

Now, using (1.9), we have

$$K_{14}(a,a,a,c_3;b,c_1,c_2,b;d,d,d,d;1,1,1,t)$$

$$=\sum_{q=0}^{\infty}\frac{(b)_q(c_3)_q\ t^q}{(d)_q\ q!}\times\frac{\Gamma(d+q)\Gamma(d+q-a-b-q-c_1-c_2)}{\Gamma(d+q-a)\Gamma(d+q-b-q-c_1-c_2)}$$

$$=\frac{\Gamma(d)\Gamma(d-a-b-c_1-c_2)}{\Gamma(d-a)\Gamma(d-b-c_1-c_2)}\times\sum_{q=0}^{\infty}\frac{(b)_q(c_3)_q\ t^q}{(d-a)_q\ q!}$$

$$=\frac{\Gamma(d)\Gamma(d-a-b-c_1-c_2)}{\Gamma(d-a)\Gamma(d-b-c_1-c_2)}\times {}_2F_1\left[\begin{matrix}b,c_3\,;\\ d-a\,;\end{matrix}t\right]$$

This completes the proof of (2.3).

Theorem 2.3. The following transformation formula for K_{15} holds true:

$$K_{15}(a,a,a,b_5;b_1,b_2,b_3,b_4;c,c,c,c;1,1,1,t)$$

$$=\frac{\Gamma(c)\Gamma(c-a-b_1-b_2-b_3)}{\Gamma(c-a)\Gamma(c-b_1-b_2-b_3)}$$

$$\times {}_3F_2\left[\begin{matrix}b_4,b_5,c-a-b_1-b_2-b_3\,;\\ c-a,c-b_1-b_2-b_3\,;\end{matrix}t\right] \quad (2.4)$$

Proof. From the definition (1.13) of K_{15}, we have

$$K_{15}(a,a,a,b_5;b_1,b_2,b_3,b_4;c,c,c,c;1,1,1,t)$$

$$=\sum_{q=0}^{\infty}\frac{(b_4)_q(b_5)_q\ t^q}{(c)_q\ q!}\times F_D^{(3)}[a,b_1,b_2,b_3;c+q;1,1,1].$$

Now, using (1.9), we have

$$K_{15}(a,a,a,b_5;b_1,b_2,b_3,b_4;c,c,c,c;1,1,1,t)$$

$$=\sum_{q=0}^{\infty}\frac{(b_4)_q(b_5)_q\ t^q}{(c)_q\ q!}\times\frac{\Gamma(c+q)\Gamma(c+q-a-b_1-b_2-b_3)}{\Gamma(c+q-a)\Gamma(c+q-b_1-b_2-b_3)}$$

$$=\frac{\Gamma(c)\Gamma(c-a-b_1-b_2-b_3)}{\Gamma(c-a)\Gamma(c-b_1-b_2-b_3)}$$

$$\times\sum_{q=0}^{\infty}\frac{(b_4)_q(b_5)_q(c-a-b_1-b_2-b_3)_q\ t^q}{(c-a)_q(c-b_1-b_2-b_3)_q\ q!}$$

$$=\frac{\Gamma(c)\Gamma(c-a-b_1-b_2-b_3)}{\Gamma(c-a)\Gamma(c-b_1-b_2-b_3)}\times {}_3F_2\left[\begin{matrix}b_4,b_5,c-a-b_1-b_2-b_3\,;\\ c-a,c-b_1-b_2-b_3\,;\end{matrix}t\right]$$

This completes the proof of (2.4).

Remark.2.2 On taking $b_5=c-a$ in (2.4), we obtain the following transformation formula:

Corollary 2.2. The following transformation formula for K_{15} holds true:

$$K_{15}(a,a,a,c-a;b_1,b_2,b_3,b_4;c,c,c,c;1,1,1,t)$$

$$=\frac{\Gamma(c)\Gamma(c-a-b_1-b_2-b_3)}{\Gamma(c-a)\Gamma(c-b_1-b_2-b_3)}$$

$$\times {}_2F_1\left[\begin{matrix}b_4,c-a-b_1-b_2-b_3\,;\\ c-b_1-b_2-b_3\,;\end{matrix}t\right]. \quad (2.5)$$

3. Applications

In this section, we derive certain summation formulas for Exton's quadruple hypergeometric functions K_{11}, K_{14} and K_{15} as applications of the results derived in the previous section.

1. Taking $t=1$, $b_4=c-b_1-b_2-b_3$, $d=c$ in (2.1) and using Dixon's summation theorem (1.8), we get

$$K_{11}(a,a,a,a;b_1,b_2,b_3,c-b_1-b_2-b_3;c,c,c,c;1,1,1,1)$$

$$=\frac{\Gamma(c)\Gamma(c-a-b_1-b_2-b_3)}{\Gamma(c-a)\Gamma(c-b_1-b_2-b_3)}$$

$$\times\frac{\Gamma(1+\frac{1}{2}a)\Gamma(c)\Gamma(1+a-c+b_1+b_2+b_3)\Gamma(b_1+b_2+b_3-\frac{1}{2}a)}{\Gamma(1+a)\Gamma(1+\frac{1}{2}a-c+b_1+b_2+b_3)\Gamma(c-\frac{1}{2}a)\Gamma(b_1+b_2+b_3)}. \quad (3.1)$$

2. Taking $t=1$ in (2.2) and using Gauss's summation theorem (1.4), we get

$$K_{11}\left(a,a,a,a;b_1,b_2,b_3,1+a-c+b_1+b_2+b_3;c,c,c,d;1,1,1,1\right)$$

$$=\frac{\Gamma(c)\Gamma(c-a-b_1-b_2-b_3)\Gamma(d)\Gamma(d+c-2a-1)}{\Gamma(c-a)\Gamma(c-b_1-b_2-b_3)\Gamma(d-a)\Gamma(d+c-a-1)}. \quad (3.2)$$

3. Taking $t=\frac{1}{2}$, $d=1+a-\frac{1}{2}c$ in (2.2) and using Gauss's second summation theorem (1.5), we get

$$K_{11}\left(a,a,a,a;b_1,b_2,b_3,1+a-c+b_1+b_2+b_3;c,c,c,1+a-\tfrac{1}{2}c;1,1,1,\tfrac{1}{2}\right)$$

$$=\frac{\Gamma(c)\Gamma(c-a-b_1-b_2-b_3)\Gamma(\frac{1}{2})\Gamma(1+a-\frac{1}{2}c)}{\Gamma(c-a)\Gamma(c-b_1-b_2-b_3)\Gamma(\frac{1}{2}a+\frac{1}{2})\Gamma(1+\frac{1}{2}a-\frac{1}{2}c)}. \quad (3.3)$$

4. Taking $t=\frac{1}{2}, c=2a$ in (2.2) and using Bailey's summation theorem (1.6), we get

$$K_{11}\left(a,a,a,a;b_1,b_2,b_3,1+a-c+b_1+b_2+b_3;2a,2a,2a,d;1,1,1,\tfrac{1}{2}\right)$$

$$=\frac{\Gamma(2a)\Gamma(a-b_1-b_2-b_3)\Gamma(\frac{1}{2}d)\Gamma(\frac{1}{2}d+\frac{1}{2})}{\Gamma(a)\Gamma(2a-b_1-b_2-b_3)\Gamma(\frac{1}{2}d+\frac{1}{2}a)\Gamma(\frac{1}{2}d-\frac{1}{2}a+\frac{1}{2})}. \quad (3.4)$$

5. Taking $t=-1$, $d=c$ in (2.2) and using Kummer's summation theorem (1.7), we get

$$K_{11}\left(a,a,a,a;b_1,b_2,b_3,1+a-c+b_1+b_2+b_3;c,c,c,c;1,1,1,-1\right)$$
$$=\frac{\Gamma(c)\Gamma(c-a-b_1-b_2-b_3)\Gamma(c)\Gamma(1+\frac{1}{2}a)}{\Gamma(c-a)\Gamma(c-b_1-b_2-b_3)\Gamma(1+a)\ \Gamma(c-\frac{1}{2}a)}. \quad (3.5)$$

6. Taking $t=1$ in (2.3) and using Gauss's summation theorem (1.4), we get

$$K_{14}\left(a,a,a,c_3;b,c_1,c_2,b;d,d,d,d;1,1,1,1\right)$$

$$=\frac{\Gamma(d)\Gamma(d-a-b-c_1-c_2)\Gamma(d-a-b-c_3)}{\Gamma(d-b-c_1-c_2)\Gamma(d-a-b)\Gamma(d-a-c_3)}. \quad (3.6)$$

7. Taking $t=\frac{1}{2}$, $d=\frac{1}{2}\left(2a+b+c_3+1\right)$ in (2.3) and using Gauss's second summation theorem (1.5), we get

$$K_{14}(a,a,a,c_3;b,c_1,c_2,b;\tfrac{1}{2}(2a+b+c_3+1),\tfrac{1}{2}(2a+b+c_3+1),$$

$$\tfrac{1}{2}(2a+b+c_3+1),\tfrac{1}{2}(2a+b+c_3+1),\tfrac{1}{2}(2a+b+c_3+1);1,1,1,\tfrac{1}{2})$$

$$=\frac{\Gamma(\frac{1}{2})\Gamma(a+\frac{1}{2}b+\frac{1}{2}c_3+\frac{1}{2})\Gamma(\frac{1}{2}-\frac{1}{2}b+\frac{1}{2}c_3-c_1-c_2)}{\Gamma(\frac{1}{2}b+\frac{1}{2})\Gamma(\frac{1}{2}c_3+\frac{1}{2})\Gamma(a-\frac{1}{2}b+\frac{1}{2}c_3-c_1-c_2+\frac{1}{2})}. \quad (3.7)$$

Further, taking $c_3=a$ in (3.7), we get

$$K_{14}(a,a,a,a;b,c_1,c_2,b;\tfrac{1}{2}(3a+b+1),\tfrac{1}{2}(3a+b+1),$$

$$\tfrac{1}{2}(3a+b+1),\tfrac{1}{2}(3a+b+1);1,1,1,\tfrac{1}{2})$$

$$=\frac{\Gamma(\frac{1}{2})\Gamma(\frac{3}{2}a+\frac{1}{2}b+\frac{1}{2})\Gamma(\frac{1}{2}a-\frac{1}{2}b-c_1-c_2+\frac{1}{2})}{\Gamma(\frac{1}{2}a+\frac{1}{2})\Gamma(\frac{1}{2}b+\frac{1}{2})\Gamma(\frac{3}{2}a-\frac{1}{2}b-c_1-c_2+\frac{1}{2})}. \quad (3.8)$$

8. Taking $t=\frac{1}{2}, c_3=1-b$ in (2.3) and using Bailey's summation theorem (1.6), we get

$$K_{14}\left(a,a,a,1-b;b,c_1,c_2,b;d,d,d,d;1,1,1,\tfrac{1}{2}\right)$$

$$=\frac{\Gamma(d)\Gamma(d-a-b-c_1-c_2)\Gamma(\frac{1}{2}d-\frac{1}{2}a)\Gamma(\frac{1}{2}d-\frac{1}{2}a+\frac{1}{2})}{\Gamma(d-a)\Gamma(d-b-c_1-c_2)\Gamma(\frac{1}{2}d-\frac{1}{2}a+\frac{1}{2}b)\Gamma(\frac{1}{2}d-\frac{1}{2}a-\frac{1}{2}b+\frac{1}{2})}. \quad (3.9)$$

9. Taking $t=-1$, $d=1+a+b-c_3$ in (2.3) and using Kummer's summation theorem (1.7), we get

$$K_{14}(a,a,a,c_3;b,c_1,c_2,b;1+a+b-c_3,1+a+b-c_3,$$

$$1+a+b-c_3,1+a+b-c_3;1,1,1,-1)$$

$$=\frac{\Gamma(1+a+b-c_3)\Gamma(1-c_1-c_2-c_3)\Gamma(1+\frac{1}{2}b)}{\Gamma(1+a-c_1-c_2-c_3)\Gamma(1+\frac{1}{2}b-c_3)\Gamma(1+b)}. \quad (3.10)$$

Further, taking $c_3=a$ in (3.10), we get

$$K_{14}\left(a,a,a,a;b,c_1,c_2,b;1+b,1+b,1+b,1+b;1,1,1,-1\right)$$

$$=\frac{\Gamma(1-a-c_1-c_2)\Gamma(1+\frac{1}{2}b)}{\Gamma(1-c_1-c_2)\Gamma(1+\frac{1}{2}b-a)}. \quad (3.11)$$

10. Taking $t=1$, $b_4=1-a$, $b_5=1-b_1-b_2-b_3$ in (2.4) and using Dixon's summation theorem (1.8), we get

$$K_{15}\left(a,a,a,1-b_1-b_2-b_3;b_1,b_2,b_3,1-a;c,c,c,c;1,1,1,1\right)$$

$$=\frac{\Gamma(c)\Gamma(c-a-b_1-b_2-b_3)\Gamma(1+\frac{1}{2}c-\frac{1}{2}a-\frac{1}{2}b_1-\frac{1}{2}b_2-\frac{1}{2}b_3)}{\Gamma(c-1)\Gamma(1+c-a-b_1-b_2-b_3)\Gamma(\frac{1}{2}c+\frac{1}{2}a-\frac{1}{2}b_1-\frac{1}{2}b_2-\frac{1}{2}b_3)}$$

$$\times\frac{\Gamma(\frac{1}{2}c+\frac{1}{2}a+\frac{1}{2}b_1+\frac{1}{2}b_2+\frac{1}{2}b_3-1)}{\Gamma(\frac{1}{2}c-\frac{1}{2}a+\frac{1}{2}b_1+\frac{1}{2}b_2+\frac{1}{2}b_3)}. \quad (3.12)$$

11. Taking $t=1$ in (2.5) and using Gauss's summation theorem (1.4), we get

$$K_{15}\left(a,a,a,c-a;b_1,b_2,b_3,b_4;c,c,c,c;\ 1,\ 1,\ 1,\ 1\right)$$

$$=\frac{\Gamma(c)\Gamma(c-a-b_1-b_2-b_3)\Gamma(a-b_4)}{\Gamma(c-a)\Gamma(c-b_1-b_2-b_3-b_4)\Gamma(a)}. \quad (3.13)$$

12. Taking $t=-1, b_4=1-a$ in (2.5) and using Kummer's summation theorem (1.7), we get

$$K_{15}(a,a,a,c-a;b_1,b_2,b_3,1-a;c,c,c,c;\,1,1,1,-1)$$

$$=\frac{\Gamma(c)\Gamma(c-a-b_1-b_2-b_3)\Gamma(1+\frac{1}{2}c-\frac{1}{2}a-\frac{1}{2}b_1-\frac{1}{2}b_2-\frac{1}{2}b_3)}{\Gamma(c-a)\Gamma(1+c-a-b_1-b_2-b_3)\Gamma(\frac{1}{2}c+\frac{1}{2}a-\frac{1}{2}b_1-\frac{1}{2}b_2-\frac{1}{2}b_3)}. \quad (3.14)$$

Conclusion

In the present paper, we derived a transformation formulas for Exton's quadruple hypergeometric functions K_{11}, K_{14} and K_{15}. Furthermore, as an applications of our main formula, we have present certain summation formulas for K_{11}, K_{14} and K_{15}. The results are derived by using the method of series manipulation with the help of the well-known classical summation theorems. The method used in this paper can be applied to derive certain transformation and summation formulas for other hypergeometric functions given in the literature.

Acknowledgements

The authors are thankful to the referees for useful comments and suggestions towards the improvement of this paper.

References

[1] J. Choi, A. K. Rathie and Purnima, “A Note on Gauss's second summation theorems for the series ${}_2F_1(\frac{1}{2})$ ”, Commun. Korean Math. Soc. vol. 22, no. 4, p. 509-512, (2007).

[2] H. Exton, “Certain hypergeometric functions of four variables”. Bull. Soc. Math. Gréce (N.S.) vol. 13, p. 104-113, (1972).

[3] H. Exton, “Multiple hypergeometric functions and applications”, Halsted Press, New York, (1976).

[4] Y. S. Kim, A. K. Rathie and J. Choi, “Note on Srivastava's triple hypergeometric series H_C ”, Commun. Korean Math. Soc. vol. 18, no. 3, p. 581-586, (2003).

[5] E. D. Rainville, “Special Functions” (Macmillan Company, New York, (1960); Reprintedby Chelsea Publishing Company, Bronx, New York, 1971).

[6] Salahuddin, M.P. Chaudhary and A. Arora, “Some summation theorems involving Bailey theorem”, Global Journal of Science Frontier Research, vol. 12, no. 9, p. 71-97, (2012).

[7] H. M. Srivastava and H. L. Manocha, “A Treatise on Generating Functions”, Halsted Press, New York, (1984).

مقالة بحثية

تحويلات وصيغ جمعية لدوال اكستون K_{11}، K_{14} و K_{15}

أحمد علي عتش[1،*]، حسين صالح بلحاج[2]

[2،1] قسم الرياضيات، كلية التربية - شبوة، جامعة عدن، عدن، اليمن

[*] الباحث الممثل: أحمد علي عتش بريد الكتروني ah-a-atash@hotmail.com

استلم في: 31 يناير 2020 / قبل في: 18 فبراير 2020 / نشر في: 07 مارس 2020

الملخص

هدف بحثنا هذا هو اثبات بعض التحويلات لدوال اكستون الفوق هندسية الرباعية K_{11}، K_{14} و K_{15}. وكتطبيقات لنتائج بحثنا الرئيسية تم ايضا اشتقاق العديد من صيغ الجمع ل K_{11}، K_{14} و K_{15} وذلك بمساعدة نظريات الجمع الكلاسيكية.

الكلمات الرئيسية: تحويلات، صيغ جمعية، الدوال الفوق هندسية، نظريات الجمع الكلاسيكية.