

RESEARCH ARTICLE

PROPAGATION OF ION ACOUSTIC WAVES IN A MAGNETIZED QUANTUM PLASMA IN THE PRESENCE OF EXCHANGE-CORRELATION EFFECTS

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Abstract

The nonlinear propagation of ion acoustic solitary waves are studied in a magnetized quantum plasma consisting of cold inertia ions and inertialless quantum electrons and positrons, including exchange-correlation effects. A Zakharov-Kuznetsov equation is derived by using the reductive perturbation method. The effects of quantum plasma parameters on the propagation characteristics of the ion acoustic solitary waves have been investigated. It is found that the phase velocity, amplitude and width of the solitary waves are significantly affected by the presence of exchange-correlation potentials of electron and positron. Only solitary wave width effected by both quantum diffraction and magnetic field strength. The width of the solitary waves increases with the increase of both the quantum diffraction and magnetic field strength. The increase in the positron concentration causes to diminish both the solitary waves amplitude and width. The current results may be useful to understand the properties of ion acoustic waves propagating in dense space plasma environments where the quantum effects are expected to dominate.

Keywords: Ion-acoustic waves, Quantum plasma, Exchange-correlation.

1. Introduction

The electron-positron (EP) plasma is believed to exist in a pulsar magnetosphere [1, 2], in bipolar outflows, in active galactic nuclei [3], and in the early universe [4]. Though dominant constituents of these astrophysical plasma are electrons and positron but in the atmosphere around astrophysical objects, beside electron-positron pairs a small number of heavy ions is also likely to be present [5]. For example, the magnetosphere of the neutron stars is filled with electron-positron plasma, however, it is believed that it may have some fraction of ions as well. The presence of some fraction of ions in the neutron star magnetosphere is assumed to be originated from some interior source such as a result of evaporation or seismic processes on the surface of neutron star. The ions can also enter in the magnetosphere of neutron/pulsar from outside in the process of accretion [6]. Accordingly, it is important to study the dynamics of the nonlinear wave motions in an electron-positron-ion (EPI) plasma. During the last three decades, EP and EPI plasmas have attracted significant attention among

researchers [7-15]. Some of these investigations deal with the nonlinear ion acoustic waves (IAWs) in the framework of classical plasmas. On the other hand, quantum plasmas have drawn attention of many researchers due to its applications in different environments, e.g. in super-dense astrophysical objects [16] (such as the interior of Jupiter and massive white dwarfs, magnetars, and neutron stars), in high-intensity laser-produced plasmas [17, 18], and in ultra-small electronic devices [19], quantum dots, nanowires [20], carbon nanotubes [21], quantum diodes [22, 23], biophotonics [24], ultra-cold plasmas [25], and micro-plasmas [26].

One of the important properties of dense quantum plasmas is that the plasma particles can be subject to new significant quantum forces, one of them is the gradient force of quantum Bohm potential [27] which arises due to the separation of charges in a plasma. Another important force arises due to the exchange-correlation effects of dense plasma particles [28] where the interaction of quantum particles can be separated into a

Hartree term due to electrostatic potential of electron/positron number density and the exchange-correlation term due to the spin effect. Rahman et al. [29] investigated small but finite amplitude electrostatic solitary waves in a relativistically degenerate dense magneto-plasma and derived a Zakharov-Kuznetsov (ZK) equation by using the reductive perturbation technique. Sahu et al. [30] studied the oblique propagation of IAWs in a magnetized degenerate dense magneto-plasma. They assumed that the plasma is to be rotating with angular frequency at an angle θ to the direction of the magnetic field. Paul et al. [31] studied the nonlinear propagation of ion acoustic waves in unmagnetized quantum plasma in the presence of an ion beam using the one-dimensional quantum hydrodynamic model. They have found that the formation and structure of solitary waves are significantly affected by the ion beam.

However, the propagation of ion acoustic solitary waves in quantum plasma in the presence of electron/positron exchange-correlation potential are not studied yet. Therefore, the aim of the current paper is to investigate the nonlinear propagation of quantum ion acoustic waves (IAWs) in a degenerate EPI plasma in the presence of quantum Bohm potential and exchange-correlation potential of electrons and positrons. Also, we consider the electrons and positrons are obey the degeneracy pressure law (Fermi pressure), while the inertial ions are taken to be cold and magnetized. The plasma is assumed to be embedded in a constant external magnetic field pointing in the z-direction.

This paper is organized as follows: the basic equations governing the quantum magneto-plasma system under consideration are presented in Sect. 2. In Sect. 3, a Zakharov-Kuznetsov (ZK) equation is derived using the reductive perturbation method. The solitary wave solution of the ZK equation and stability analysis are obtained in Sect. 4. The properties of the electrostatic solitary potential are discussed in Sect. 5. Finally, Sect. 6 is kept for conclusion.

2. Governing equations

Let us consider a collisionless magnetized EPI quantum plasma composed of inertial positively charged ions and inertialless degenerated electrons and positrons, including exchange-correlation effects. In dense astrophysical environments, the Fermi pressure for the ions is negligible as compared to that for the electrons and positrons. So the pressure effects are neglected for the ions, whereas the electrons and positrons are assumed to obey the equation of state for a zero temperature Fermi gas. At equilibrium, we have the charge neutrality condition as $n_{e0} = n_{p0} + n_{i0}$, where n_{e0} , n_{p0} and n_{i0} are the unperturbed number densities of electrons, positrons and ions, respectively. We suppose that the

plasma model under consideration is subjected to external magnetic field of strength B_0 along the z-axis i.e. $\mathbf{B} = B_0 \mathbf{e}_z$, where \mathbf{e}_z is the unit vector along the z-axis. The nonlinear dynamics of IAWs propagating in such quantum plasma model are governed by the following equations

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{u}_i) = 0, \quad (1)$$

$$\frac{\partial \mathbf{u}_i}{\partial t} + (\mathbf{u}_i \cdot \nabla) \mathbf{u}_i = -\frac{e}{m_i} \nabla \phi + \frac{e}{m_i} (\mathbf{u}_i \times \mathbf{B}) \quad (2)$$

$$0 = -q_j \nabla \phi - \frac{1}{n_j} \nabla P_{Fj} + \frac{\hbar^2}{2m} \nabla \left(\frac{1}{\sqrt{n_j}} \nabla^2 \sqrt{n_j} \right) - \nabla V_j^{xc}, \quad (3)$$

$$\nabla^2 \phi = \frac{e}{\epsilon_0} (n_e - n_i - n_p), \quad (4)$$

where $n_i(\mathbf{u}_i)$ is the number density (fluid velocity) of ions, n_j is the number density of electrons or positrons, m_i is the ion mass, $m = m_e = m_p$ is the electrons/positrons mass and ϕ is electrostatic potential. Here, the subscript $j = e$ for electrons and $j = p$ for positrons, q_j is the charge of an electron or a positron i.e. $q_e = -e$ for electrons and $q_p = e$ for positrons where e is the electronic charge.

The two term in the right-hand side of Eq. (3) is due to the degenerate Fermi pressure P_{Fj} of electrons or positrons, which is given by

$$P_{Fj} = \frac{2E_{Fj}n_{j0}}{5} \left(\frac{n_j}{n_{j0}} \right)^{5/3}, \quad (5)$$

where $E_{Fj} (= k_B T_{Fj})$ is the Fermi energy and $T_{Fj} = \hbar^2 (3\pi^2 n_{j0})^{2/3} / 2m k_B$ is the Fermi temperature of electrons or positrons. The third term in Eq. (3) represents the gradient of the Bohm potential (corresponding to the quantum tunneling effect), and the last term is the gradient of the exchange-correlation potential of the degenerate plasma particles (electrons or positrons), which is given by [28, 32]

$$V_j^{xc} = - \left(\frac{0.985e^2}{4\pi\epsilon_0} \right) n_j^{1/3} \left[1 + \frac{0.034}{a_B n_j^{1/3}} \ln(1 + 18.367 a_B n_j^{1/3}) \right], \quad (6)$$

where $a_B = 4\pi\epsilon_0 \hbar^2 / me^2$ is the Bohr radius. In the dense plasma, the condition $18.37 a_B n_j^{1/3} \ll 1$ is satisfied, thus, the exchange-correlation potential V_j^{xc} can be approximated as

$$V_j^{xc} \approx -1.6 \left(\frac{e^2}{4\pi\epsilon_0} \right) n_j^{1/3} + 5.62 \left(\frac{\hbar^2}{m} \right) n_j^{2/3}, \quad (7)$$

To simplify, all physical quantities appearing in the above equations are to be appropriately normalized. Accordingly, we normalize as follows: $N_s = n_s/n_{s0}$, $\mathbf{U}_i = \mathbf{u}_i/C_i$, and $\phi = \phi/(E_{Fe}/e)$, where $C_i = (2k_B T_{Fe}/m_i)^{1/2}$ is the speed of ion acoustic waves and $\mathbf{U}_i = (U_{ix}, U_{iy}, U_{iz})$, with U_{ix} , U_{iy} and U_{iz} are the velocity of ions in x , y , and z directions, respectively. The time and space variables are normalized as $t \rightarrow \omega_{pi} t$

and $r \rightarrow (\omega_{pi}/C_i)r$, respectively where $\omega_{pi} = (e^2 n_{i0}/\epsilon_0 m_i)^{1/2}$ is the ion plasma frequency. Accordingly, we can rewrite the normalized basic equations which describe the three dimensional propagation of quantum ion acoustic solitary waves (IASWs) as

$$\frac{\partial N_i}{\partial t} + \frac{\partial N_i U_{ix}}{\partial x} + \frac{\partial N_i U_{iy}}{\partial y} + \frac{\partial N_i U_{iz}}{\partial z} = 0, \quad (8)$$

$$\frac{\partial U_{ix}}{\partial t} + U_{ix} \frac{\partial U_{ix}}{\partial x} + U_{iy} \frac{\partial U_{ix}}{\partial y} + U_{iz} \frac{\partial U_{ix}}{\partial z} = -\frac{\partial \phi}{\partial x} + \Omega_{ci} U_{iy}, \quad (9)$$

$$\frac{\partial U_{iy}}{\partial t} + U_{ix} \frac{\partial U_{iy}}{\partial x} + U_{iy} \frac{\partial U_{iy}}{\partial y} + U_{iz} \frac{\partial U_{iy}}{\partial z} = -\frac{\partial \phi}{\partial y} - \Omega_{ci} U_{ix}, \quad (10)$$

$$\frac{\partial U_{iz}}{\partial t} + U_{ix} \frac{\partial U_{iz}}{\partial x} + U_{iy} \frac{\partial U_{iz}}{\partial y} + U_{iz} \frac{\partial U_{iz}}{\partial z} = -\frac{\partial \phi}{\partial z}, \quad (11)$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = \frac{1}{1-p} N_e - \frac{p}{1-p} N_p - N_i, \quad (12)$$

where $p = n_{p0}/n_{e0}$, and $\Omega_{ci} = \omega_{ci}/\omega_{pi}$, $\omega_{ci} = eB/m_i$ is the ion gyro-frequency. The normalized momentum equations of electrons and positrons are given by

$$\nabla \phi + \frac{H^2}{2} \nabla \left(\frac{\nabla^2 \sqrt{N_e}}{\sqrt{N_e}} \right) - \frac{1}{2} (1 + 2\gamma) \nabla N_e^{2/3} + \alpha \nabla N_e^{1/3} = 0, \quad (13)$$

$$\nabla \phi - \frac{H^2}{2} \nabla \left(\frac{\nabla^2 \sqrt{N_p}}{\sqrt{N_p}} \right) + \frac{1}{2} (\sigma + 2p^{2/3}\gamma) \nabla N_p^{2/3} - p^{1/3} \alpha \nabla N_p^{1/3} = 0, \quad (14)$$

where $H = \omega_{pi} \hbar / \sqrt{m m_i} C_i^2$, $\sigma = T_{Fp}/T_{Fe}$, $\alpha = 1.6(e^2 n_{e0}^{1/3} / 8\pi \epsilon_0 E_{Fe})$, and $\gamma = 5.65(\hbar^2 n_{e0}^{2/3} / 2mE_{Fe})$.

3. Derive Zakharov-Kuznetsov equation

In order to derive a Zakharov-Kuznetsov (ZK) equation, we employ the standard reductive perturbation technique (RPT) [33]. Accordingly, we introduce the following stretched coordinates

$$X = \sqrt{\epsilon}x, Y = \sqrt{\epsilon}y, Z = \sqrt{\epsilon}(z - \lambda_0 t), \tau = \sqrt{\epsilon^3}t, \quad (15)$$

where ϵ is a small parameter ($0 < \epsilon < 1$) measuring the strength of the nonlinearity, and λ_0 is the linear phase velocity of IAW normalized by C_i .

All dependent variables appearing in the Eqs. (8)-(14) are expanded about their equilibrium as a power series of ϵ as

$$\begin{pmatrix} N_s \\ U_{iz} \\ \phi \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \epsilon \begin{pmatrix} N_{s1} \\ U_{iz1} \\ \phi_1 \end{pmatrix} + \epsilon^2 \begin{pmatrix} N_{s2} \\ U_{iz2} \\ \phi_1 \end{pmatrix} + \dots, s = i, e, p \quad (16)$$

and

$$U_{ir} = \epsilon^{3/2} U_{ir1} + \epsilon^2 U_{ir2} + \dots, r = x, y. \quad (17)$$

Substituting Eqs. (15)-(17) into the Eqs. (8)-(14), and then collecting the terms of like powers of ϵ , in lowest order, we obtain the following relations:

$$N_{i1} = \frac{1}{\lambda_0^2} \phi_1, \quad (18)$$

$$U_{iz1} = \frac{1}{\lambda_0} \phi_1, \quad (19)$$

$$N_{e1} = \frac{3}{1-\alpha_e} \phi_1, N_{p1} = -\frac{3}{\sigma-\alpha_p} \phi_1, \quad (20)$$

$$N_{e1} - pN_{p1} - (1-p)N_{i1} = 0, \quad (21)$$

where $\alpha_e = \alpha - 2\gamma$, and $\alpha_p = p^{1/3}(\alpha - 2p^{1/3}\gamma)$.

Now, substituting Eqs. (18) and (20) into the Eq. (21), we get the linear phase velocity of the IAW as

$$\lambda_0 = \sqrt{\left(\frac{1-p}{3}\right) \frac{(1-\alpha_e)(\sigma-\alpha_p)}{\sigma-\alpha_p+p(1-\alpha_e)}}. \quad (22)$$

It can be noted from Eq. (22) that the presence of the exchange-correlation potential (via the parameters α_e , α_p or α , γ) significantly modifies the linear phase velocity of IASWs. Similarly, we write first order x and y -components of ion momentum equations as

$$U_{ix1} = -\frac{1}{\Omega_{ci}} \frac{\partial \phi_1}{\partial Y}, \quad (23)$$

$$U_{iy1} = \frac{1}{\Omega_{ci}} \frac{\partial \phi_1}{\partial X}. \quad (24)$$

To the next higher-order of ϵ , we obtain the second order x - and y -component of ion momentum equations as

$$U_{ix2} = \frac{\lambda_0}{\Omega_{ci}^2} \frac{\partial^2 \phi_1}{\partial Z \partial X}, \quad (25)$$

$$U_{iy2} = \frac{\lambda_0}{\Omega_{ci}^2} \frac{\partial^2 \phi_1}{\partial Z \partial Y}. \quad (26)$$

By following the same procedure, we can obtain respectively the next higher-order continuity equation, the z -component of ion momentum equation and the next higher-order Poisson equation as

$$\frac{\partial N_{i1}}{\partial \tau} - \lambda_0 \frac{\partial N_{iz2}}{\partial Z} + \frac{\partial U_{iz2}}{\partial Z} + \frac{\partial N_{i1} U_{iz1}}{\partial Z} + \frac{\partial U_{ix2}}{\partial X} + \frac{\partial U_{iy2}}{\partial Y} = 0, \quad (27)$$

$$\frac{\partial U_{iz1}}{\partial \tau} - \lambda_0 \frac{\partial U_{iz2}}{\partial Z} + U_{iz1} \frac{\partial U_{iz1}}{\partial Z} + \frac{\partial \phi_2}{\partial Z} = 0, \quad (28)$$

$$\frac{\partial^2 \phi_1}{\partial X^2} + \frac{\partial^2 \phi_1}{\partial Y^2} + \frac{\partial^2 \phi_1}{\partial Z^2} = \frac{1}{1-p} N_{e2} - \frac{p}{1-p} N_{p2} - N_{iz2}. \quad (29)$$

Here, the second order momentum equations of electrons and positrons give

$$N_{e2} = \frac{9H^2}{4(1-\alpha_e)^2} \nabla^2 \phi_1 + \frac{3}{(1-\alpha_e)} \phi_2 + \frac{3[1+2(\gamma-\alpha)]}{2(1-\alpha_e)^3} \phi_1^2, \quad (30)$$

$$N_{p2} = \frac{9H^2}{4(\sigma-\alpha_p)^2} \nabla^2 \phi_1 - \frac{3}{(\sigma-\alpha_p)} \phi_2 + \frac{3[\sigma+2p^{1/3}(p^{1/3}\gamma-\alpha)]}{2(\sigma-\alpha_p)^3} \phi_1^2, \quad (31)$$

where $\nabla^2 = \partial^2/\partial X^2 + \partial^2/\partial Y^2 + \partial^2/\partial Z^2$. Solving the system of Eqs. (27)–(31) with the aid of Eqs. (18)–(26), we get finally the following nonlinear partial differential equation

$$\frac{\partial \phi_1}{\partial \tau} + A \phi_1 \frac{\partial \phi_1}{\partial Z} + B \frac{\partial^3 \phi_1}{\partial Z^3} + D \frac{\partial}{\partial Z} \left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} \right) \phi_1 = 0. \quad (32)$$

Equation (32) is a Zakharov-Kuznetsov equation describing nonlinear propagation of three dimensions IASWs in our model, in which the nonlinear coefficient A is given as

$$A = \frac{3}{2\lambda_0} + \frac{3\lambda_0^3}{2(1-p)} \left\{ \frac{p[\sigma - 2p^{1/3}(\alpha - p^{1/3}\gamma)]}{(\sigma - \alpha_p)^3} - \frac{1 + 2(\gamma - \alpha)}{(1 - \alpha_e)^3} \right\},$$

and dispersion coefficients B and D are respectively given by

$$B = \frac{\lambda_0^3}{2} + \frac{9H^2\lambda_0^3}{8(1-p)} \left[\frac{p}{(\sigma - \alpha_p)^2} - \frac{1}{(1 - \alpha_e)^2} \right],$$

$$D = B + \frac{\lambda_0^3}{2\Omega_{ci}^2}.$$

4. Solitary wave solution of the ZK equation and stability analysis

4.1. Solitary wave solution

To obtain explicit traveling wave solutions of quantum ZK equation (32), we introduce the following traveling wave transformation:

$$\phi_1(X, Y, Z, \tau) = \phi_1(\xi), \quad \xi = l_x X + l_y Y + l_z Z - u_0 \tau, \quad (33)$$

where ξ is the transformed coordinate in the co-moving frame with speed u_0 . Here, l_x , l_y and l_z are the directional cosine of the wave vector along the X , Y and Z axes, respectively (satisfying $l_x^2 + l_y^2 + l_z^2 = 1$). Now, Applying Eq. (33) to Eq. (32) and integrating once with the boundary conditions: $\phi_1, d\phi_1/d\xi, d^2\phi_1/d\xi^2 \rightarrow 0$ as $\xi \rightarrow \infty$, we get

$$-u_0\phi_1 + A_l\phi_1^2 + B_l\frac{d^2\phi_1}{d\xi^2} = 0, \quad (34)$$

where $A_l = A l_z/2$ and $B_l = l_z[B l_z^2 + D(l_x^2 + l_y^2)]$. The one-solitary wave solution of Eq. (34) is given by

$$\phi_1 = \phi_m \operatorname{sech}^2\left(\frac{\xi}{W}\right), \quad (35)$$

where $\phi_m = 3u_0/l_z A$ is the amplitude, and $W = 2\sqrt{B_l/u_0}$ is the width of the quantum IASWs. Using the relation $E_1 = -\nabla\phi_1$ with Eq. (35), the normalized electric field of the obliquely propagating three-dimensional quantum IASWs becomes

$$E_1 = 3\left(\frac{u_0}{l_z}\right)^{3/2} \frac{\tanh(\xi/W) \operatorname{sech}^2(\xi/W)}{A\sqrt{B_l l_z^2 + D(1 - l_z^2)}}, \quad (36)$$

4.2. Stability analysis

In order to determine the stability or the properties of the instability associated with a given plasma equilibrium; we shall use a method based on energy considerations. According to this method it is necessary to calculate the change in potential energy of the plasma as a result of a given perturbation. To this end, we multiply both sides of Eq. (34) by $d\phi_1/d\xi$, and then integrating once with taking into account the boundary conditions: $\phi_1,$

$d\phi_1/d\xi, d^2\phi_1/d\xi^2 \rightarrow 0$ as $\xi \rightarrow \infty$, we obtain the energy equation:

$$\frac{1}{2}\left(\frac{d\phi_1}{d\xi}\right)^2 + \Psi(\phi_1) = 0, \quad (37)$$

where $\Psi(\phi_1)$ represents the potential energy (or Sagdeev potential), which is given by

$$\Psi(\phi_1) = -\frac{u_0}{2B_l}\phi_1^2 + \frac{A_l}{3B_l}\phi_1^3. \quad (38)$$

For the existence of solitary wave solution of Eq. (32), the condition $d^2\Psi(\phi_1)/d\phi_1^2 < 0$ must satisfy at $\phi = 0$. From Eq. (38) we have

$$\left.\frac{d^2\Psi(\phi_1)}{d\phi_1^2}\right|_{\phi_1=0} = -\frac{u_0}{2B_l}. \quad (39)$$

It is clear from the Eq. (39) that, the stable solitary wave solutions will exist when $u_0/2B_l > 0$; otherwise stable solitary waves do not exist in our quantum plasma system. Since u_0 is always positive, then B_l must be greater than zero. To be $B_l > 0$, the following condition must satisfy

$$B l_z^2 + D(1 - l_z^2) > 0, \quad (40)$$

where $l_x^2 + l_y^2 = 1 - l_z^2$, and $l_z > 0$.

5. Numerical results and discussion

In this section we investigated the properties of nonlinear quantum IASWs propagating in a magnetized dense quantum plasma system consisting of cold mobile positive ions, dense quantum electrons and positrons, including exchange-correlation effect. The reductive perturbation theory is used to derive the nonlinear ZK equation (32) which is described the nonlinear IASWs in such plasma. Here, we apply our model to some typical plasma parameters found in dense astrophysical environments for electron-positron-ion quantum plasma [34]: $B_0 = (0.1 - 1) \times 10^6 T$, $n_{p0} = (0.1 - 0.9) \times 10^{30} m^{-3}$, $n_{e0} = 10^{30} m^{-3}$ and $n_{i0} = n_{e0} - n_{p0}$. Figure 1 shows how the phase velocity λ_0 of quantum IAW vary with respect to positron concentration (via the parameter $p = n_{p0}/n_{e0}$) at fixed electron concentration $n_{e0} = 10^{30} m^{-3}$. Dashed line is plotted with the presence of exchange-correlation potential effect (via the parameters $\alpha = 0.32$, and $\gamma = 0.59$), while soled line is plotted without exchange-correlation effect (via the parameters $\alpha = 0, \gamma = 0$). It is observer that the phase velocity λ_0 decrease with increasing the positron concentration $p (= n_{p0}/n_{e0})$ and the presence of the exchange-correlation effect leads to an increase in phase velocity λ_0 . Moreover, for large values of p , the effect of exchange-correlation on phase velocity λ_0 becomes less compared to small values of p .

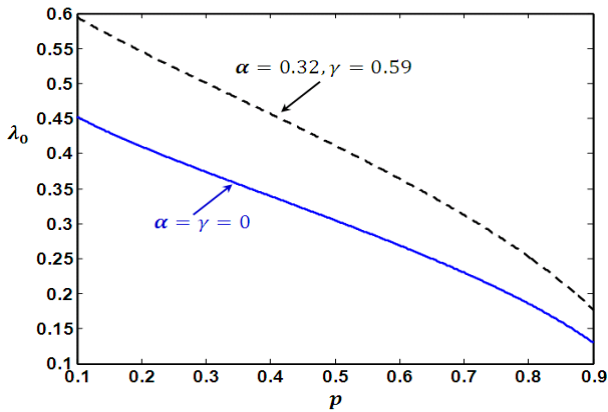


Fig.1: The variation of the phase velocity (λ_0) of IAWs against positron concentration p with (dashed line) and without (solid line) of exchange-correlation effects, alone with $n_{e0} = 10^{30}m^{-3}$, $H = 0.3216$, $B_0 = 5 \times 10^5 T$, $\Omega_{ci} = 0.0406$,

To examine the impact of exchange-correlation effects on the profile of quantum IASWs, we plot the electrostatic potential ϕ_1 versus ξ for two model, namely, in the presence (dashed line) and in the absence (solid line) of the exchange-correlation effects as shown in Fig. 2. It is noticed from Fig. 2 that, the presence of the exchange-correlation potential effects leads to an increase in both the width and amplitude quantum IASWs as depicted in Fig. 2 (dashed line). On the other hand, the shorter and narrower quantum IASWs are obtained with the absence of exchange-correlation potential as depicted in Fig. 2 (solid line).

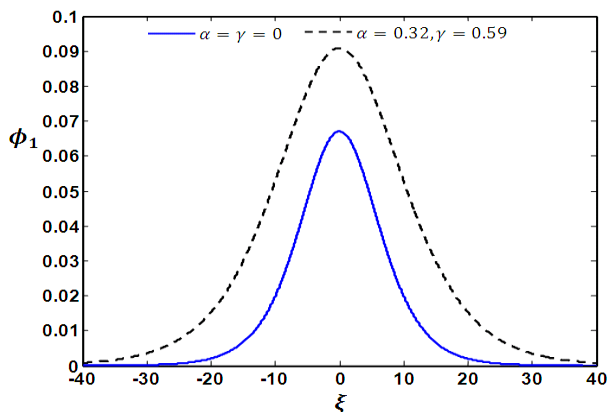


Fig.2: The profile of IASWs in the presence (dashed line) and absence (solid line) of exchange-correlation effects, with $n_{e0} = 10^{30}m^{-3}$, $\mu = 0.3$, $H = 0.3216$, $p = 0.6$, $\Omega_{ci} = 0.0406$, $l_z = 0.8$, and $u_0 = 0.1$.

To see the effect of the strength of magnetic field B_0 on the behavior of the quantum IASWs, we plot the electrostatic potential ϕ_1 versus ξ for different values of magnetic field strength B_0 as shown in Fig 3. It is noted from Fig 3 that the solitary wave width decreases as the strength of magnetic field B_0 increases. Figure 3 also indicates that the magnetic field does not affect the solitary wave amplitude. Since the normalized ion gyro-frequency Ω_{ci} is mainly associated with B_0 , ion gyro-frequency Ω_{ci} must increase with increasing the magnetic

field strength B_0 . Therefore, the solitary wave width will be decrease with ion gyro-frequency Ω_{ci} as well.

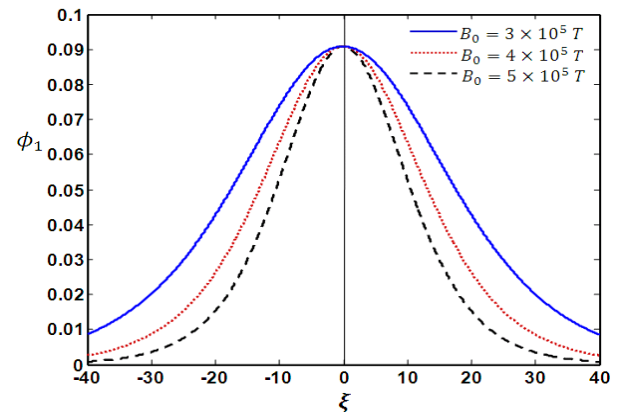


Fig.3: The profile of IASWs for different values of B_0 , with $n_{e0} = 10^{30}m^{-3}$, $n_{i0} = 0.4 \times 10^{30}m^{-3}$, $H = 0.3216$, $p = 0.6$, $l_z = 0.8$, $\alpha = 0.32$, $\gamma = 0.59$, and $u_0 = 0.1$.

Figures 4 and 5 give the variations of the amplitude ϕ_m and width W of quantum IASWs against positron concentration $p (= n_{p0}/n_{e0})$, respectively, keeping electron concentration $n_{e0} = 10^{30}m^{-3}$. It is clear from the various graphs in Figs. 4 and 5 that, the amplitude and width of the quantum IASWs decreases with increase of positron concentration p . Thus, the strength of the electrostatic potential also decreases with increase of p . The physical explanation for this is as follows: Since, increase in positron concentration reduces the ion concentration (n_{i0}) through the charge neutrality condition (i.e., $n_{i0} = n_{e0} - n_{p0}$) and since quantum IASWs are mainly associated with the ion dynamics, therefore, the width and amplitude must decrease with increasing the positron concentration. Thus, the shorter and narrower IASWs are obtained in the presence of positron concentration as compared to the IASWs without the positrons.

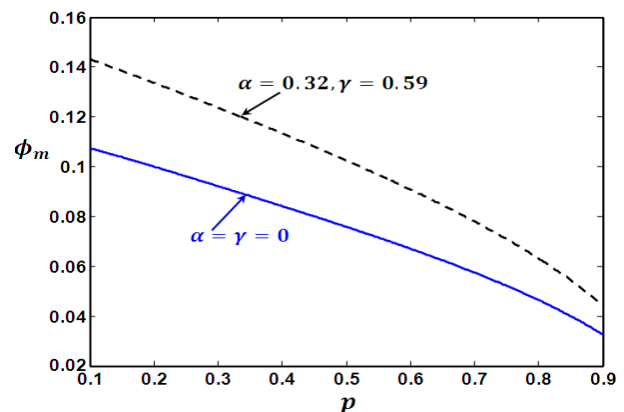


Fig.4: The variations of the solitary wave amplitude ϕ_m against positron concentration p with (dashed line) and without (solid line) of exchange-correlation effects, alone with $n_{e0} = 1 \times 10^{30}m^{-3}$, $\mu = 0.4$, $B_0 = 5 \times 10^5 T$ and $u_0 = 0.1$

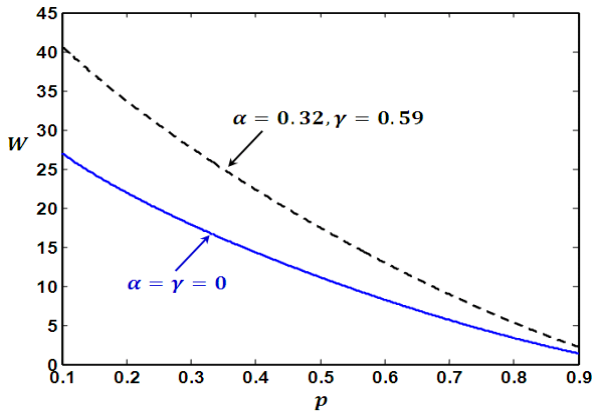


Fig.5: The variations of the solitary wave width W against positron concentration p with (dashed line) and without (solid line) of exchange-correlation effects, alone with $n_{e0} = 1 \times 10^{30} m^{-3}$, $\mu =$, $B_0 = 5 \times 10^5 T$, $l_z = 0.6$ and $u_0 = 0.1$.

Furthermore, we can see from Figs. 4 and 5 that, both amplitude ϕ_m and width W of quantum IASWs increase with the presence of exchange-correlation potential and the effects of exchange-correlation potential reduced with p , especially at larger values of positron concentration, e.g., $p = 0.9$.

Figure 6 shows the contour plot of the solitary wave amplitude ϕ_m as a function of positron concentration p and the z -component of direction cosine l_z in the presence of exchange-correlation effects. It is obvious from Fig. 6 that the solitary wave amplitude ϕ_m decreases with the increase of both p and l_z . Figure 7 shows the contour plot of the solitary wave width W as a function of p and l_z . Celery, the solitary wave width W decreases with increasing values of p . On the other hand, we can see from Fig. 7 that the width of solitary wave is enhanced when $0.1 < l_z < 0.58$ and then decreases for large values $0.58 < l_z \leq 0.9$.

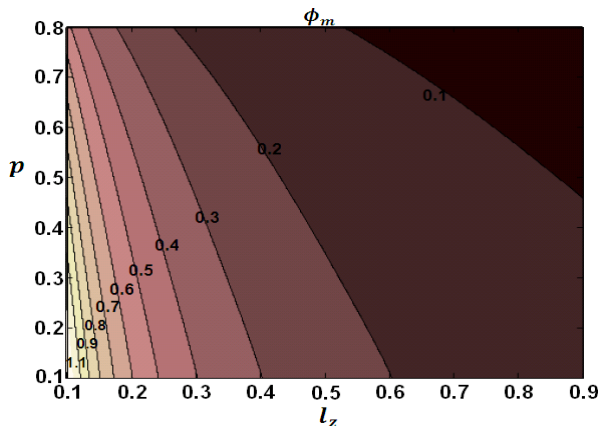


Fig.6: The variations of the solitary wave amplitude ϕ_m against the direction cosine l_z and positron concentration p , alone with $n_{e0} = 10^{30} m^{-3}$, $\mu = 0.3$, $B_0 = 10^6 T$, $\gamma = 0.59$, $\alpha = 0.148$ and $u_0 = 0.1$.

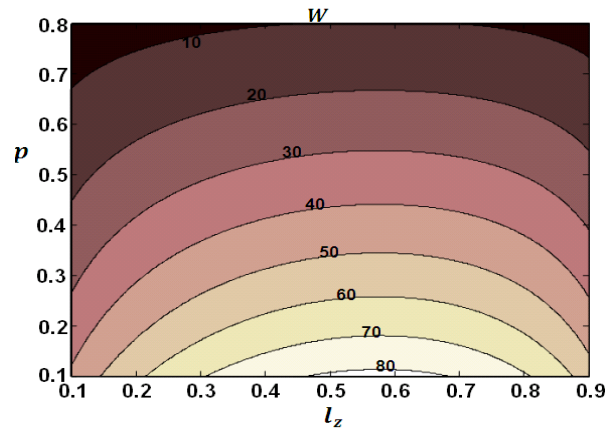


Fig.7: The variations of the solitary wave width W against the direction cosine l_z and positron concentration p , alone with $n_{e0} = 10^{30} m^{-3}$, $\mu = 0.3$, $B_0 = 10^6 T$, $\gamma = 0.59$, $\alpha = 0.148$ and $u_0 = 0.1$.

The effect of the quantum diffraction H on the solitary waves width W is depicted in Fig. 8 for different values of magnetic field strength B_0 . It is clear from this figure that the width W of the solitary waves increases with the increase of the quantum diffraction H , but decreases with magnetic field strength B_0 . For given value of quantum diffraction H , the solitary wave width decreases with the increase of magnetic field strength B_0 , and the change becomes larger with the increasing values of quantum diffraction H .

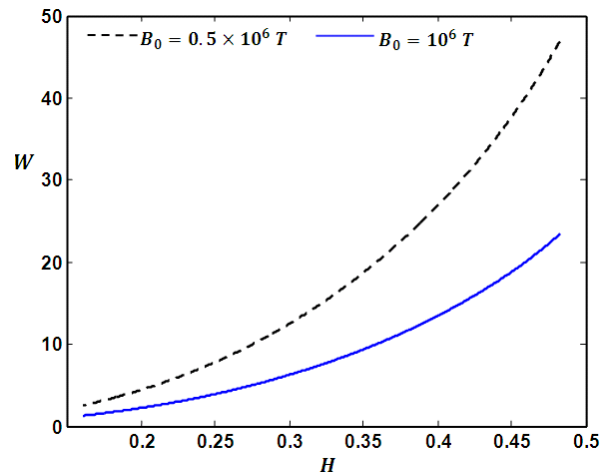


Fig.8: The variations of the solitary wave width W against the quantum diffraction H for different values of B_0 , alone with $n_{e0} = 1 \times 10^{30} m^{-3}$, $l_z = 0.6$ and $u_0 = 0.1$.

Furthermore, the behavior of the electric field E_1 profiles associated with the IASWs are presented graphically as shown in Figs. 9 and 10. Figure 9 displays the variation of electric field E_1 against ξ for different values of positron concentration p . It is observed from Fig. 9 that for small values of p , the electric field profiles spread out, and become increasingly localized with greater maximum amplitude for large values of p . Physically, this phenomena is well-understood by noting that the electric field is the negative gradient of the electrostatic

potential ϕ_1 , and hence the narrow solitary waves are obtained for high positron concentration with steeper slopes. Similarly, Fig.10 shows the variation of the electric field E_1 against ξ for different values of the direction cosine l_z . Obviously, for smaller values of $l_z < 0.6$, the electric field profiles spread out with a small finite amplitudes, but as the parameter l_z is increased in the rang $0.6 < l_z < 1$, the electric field profiles become more localized with enhanced amplitudes.

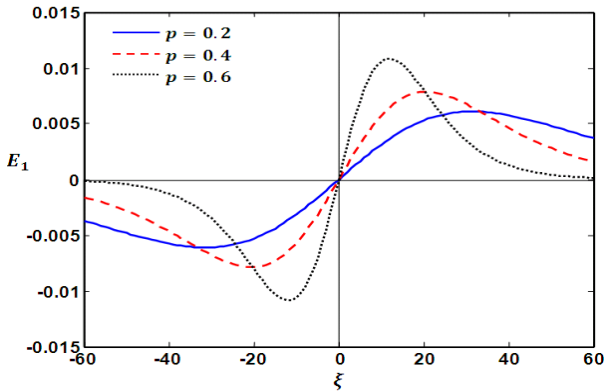


Fig.9: The electric field profiles E_1 involving IASWS for different values of positron concentration p , along with, $n_{e0} = 10^{30} m^{-3}$, $B_0 = 10^6 T$, $\alpha = 0.148$, $\gamma = 0.59$, $l_z = 0.6$ and $u_0 = 0.1$.

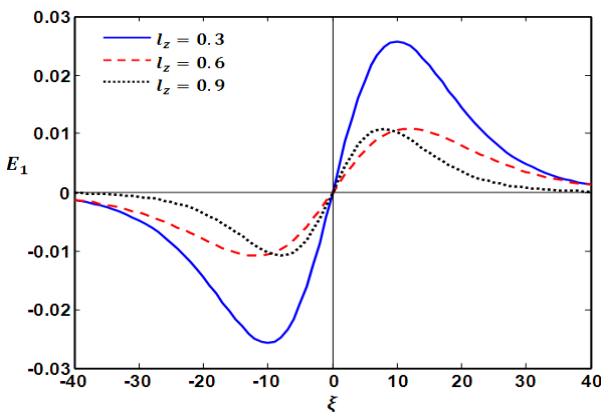


Fig.10: The electric field E_1 profiles involving IASWS for different values of the direction cosine l_z , along with $p = 0.6$, $\mu = 0.3$, $n_{e0} = 10^{30} m^{-3}$, $B_0 = 10^6 T$, $\alpha = 0.148$, $\gamma = 0.59$ and $u_0 = 0.1$.

6. Conclusions

To conclude, we have investigated the three dimensional propagation of quantum IASWs in a dense magneto-plasma, comprising of non-degenerate cold ions and dense quantum electrons and positrons. The electrons and positrons are treated to be degenerate while the cold ions are inertial and classical. By employing the reductive perturbation technique, a ZK equation is derived in terms of electrostatic potential and its solution has been analyzed. Only compressive quantum IASWs can propagate in such dense magneto-plasma. The propagation characteristics of compressive quantum IASWs are profoundly affected by the presence of

magnetic field strength B_0 , positron concentration p , the direction cosine l_z and the exchange-correlation potential (via the parameters α and γ) as well as quantum diffraction (via the parameter H). It was found that variations of p and l_z , results in the mitigation of the amplitude as well as the width of IASWs. Moreover, the increase of ion gyrofrequency Ω_{ci} (via the increase of magnetic field strength B_0) makes the solitary waves narrower with a constant amplitude. Also, the width of the IASWs increases with the quantum diffraction H . Furthermore, we observe that the effects of positron concentration p and direction cosine l_z significantly modify the associated bipolar electric field structures.

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مقالة بحثية

انتشار الموجات الصوتية الأيونية في بلازما كمية ممغنطة في وجود تأثيرات الارتباط التبادلي

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المُلخَص

تم دراسة الانتشار اللاخطي للموجات السوليتونية الصوتية الأيونية في بلازما كمية ممغنطة مكونة من أيونات قصورية باردة، والكترونات وبوزترونات كمية غير قصورية وذلك في وجود تأثيرات الترابط التبادلي للجسيمات الكمية. تم اشتقاق معادلة زخروف - كزناتسوف باستخدام طريقة الاضطراب المختزلة. تم دراسة تأثير بارامترات البلازما الكمية على خواص انتشار الموجات السوليتونية الصوتية الأيونية. من خلال الدراسة وجد أن سرعة طور الموجة وسعتها وعرضها تتأثر بشكل ملحوظ بوجود جهد الترابط التبادلي للالكترونات والبوزترونات. فقط تتأثر عرض الموجة بوجود كل من المجال المغناطيسي والانحراف الكمي. وجدنا أيضا أن زيادة كثافة البوزترونات يؤدي إلى انخفاض عرض وسعة الموجة. نتائج هذه الدراسة قد تساهم في فهم خواص الموجات الصوتية الأيونية المنتشرة في بيئات بلازما الفضاء الكثيفة والتي يكون فيها حضور للتأثيرات الكمية.

الكلمات المفتاحية: الموجات الصوتية الأيونية، البلازما الكمية، الارتباط التبادلي.

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