

RESEARCH ARTICLE

THE EFFECT OF THE WAVE FUNCTION PARAMETERS ON THE ATOM LASER DENSITY

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Abstract

This paper studies the effect of the wave function parameters on the density of ^{87}Rb atoms laser. The density equation has been derived through the use of Thomas-Fermi approximation, Green function and the stationary phase approximations. We investigated the change of density behavior with the effect of Rabi frequency, period of output coupling and detuning frequency. Consequently the number of released atoms can be controlled by determining the suitable parameters necessary for producing the best atom laser beam.

Keywords: Bose-Einstein condensation, Atom laser, Quantum optics, Matter wave.

1. Introduction

The experimental realization of a Bose-Einstein Condensation (BEC) in 1995 [1-3] has raised a lot of interest in creating coherent atomic beams using such condensates [4]. The experimental setup based on Bose Einstein condensates that produces coherent atomic beams is now called atom laser [4]. When the atoms of the condensate are extracted from a trap, they form atom laser beams [5, 6] and subsequently the size of condensate shrinks [7]. The basic operation of an atom-laser outcoupler for magnetically trapped condensates is a coupling between the trapped and untrapped states using either radio frequency (rf) or Raman transitions [8]. Most atom lasers have been produced experimentally using a radio-frequency rf coupling to produce the radio frequency atom laser [1, 9, 10]. The rf atom laser is produced by two types, a continuous atom laser which is based on continuous refilling of the condensate and a pulse atom laser whereas the condensate is periodically refilled and slowly released [11]. In this paper we deal with the quasi-continuous output which is produced by using a rf field that continuously couples atoms into a falling state due to gravity. The density of ^{87}Rb atom laser represents the main objective of this study. The wave function parameters including the Rabi frequency, the period of output coupling and detuning frequency that really affecting the density of atom laser have been studied. This study looks to be the first one carried out for these parameters.

2. Condensation Model

The main component of the atom laser is the Bose-Einstein condensate (BEC) of dilute alkali vapors which acts as the lasing mode, where the output beam is coupled to the lasing mode via transfer internal state of atom from trapped into untrapped state [12]. Considering the system consists of ^{87}Rb atoms which can develop a macroscopic population of lowest energy state under a critical temperature T_c [5]. The atoms are confined in a harmonic magnetic potential:

$$V_{trap} = \frac{1}{2}M(\omega_x^2 x^2 + \omega_{\perp}^2 y^2 + \omega_{\perp}^2 z^2). \quad (1)$$

where M is the atomic mass and ω_x, ω_{\perp} are frequencies of trap. The radio frequency magnetic field can induce transitions between states of hyperfine-manifold $F = 1$. The macroscopic wavefunctions of $2F + 1$ Zeeman sublevels are:

$$\psi_m = \psi'_m \exp[-i m \omega_{rf} t], \quad (2)$$

where $m \in \{-F, \dots, F\}$. Often in experiments, Ioffe-type traps are usually elongated in the horizontal plane being axial symmetric in the remaining directions. Considering y and z are the directions of radial confinement and x is axial confinement. In this system, the outer atoms depend on rf outcoupling can be described by Gross – Pitaevskii equations(GPE) as follows [4, 11]:

$$i\hbar \frac{\partial \psi_m(\mathbf{r}, t)}{\partial t} = \left[\frac{p^2}{2M} + V_{eff,m}(\mathbf{r}, t) \right] \psi_m(\mathbf{r}, t) + \hbar \Omega \sum_{m'} \gamma' \psi_m(\mathbf{r}, t). \quad (3)$$

where

$$V_{eff,m}(\mathbf{r}, t) = \text{sign}(g_F) \frac{1}{2} m M \omega^2 \mathbf{r}^2 + m \hbar \delta_{rf} + Mgz + U \|\psi(\mathbf{r}, t)\|^2, \quad (4)$$

$\gamma' = (\gamma_{m,m'+1} + \gamma_{m,m'-1})$ and γ is delta function. An energy term, $U \|\psi(\mathbf{r}, t)\|^2$ is expressed as (twice) the mean-field energy of the system and plays a fundamental role in its dynamics with the interaction coupling constant $U = 4\pi\hbar^2 N a/M$, where a is the s-wave scattering length and N is initial number of atoms in the system. The total atomic density divided by N is indicated by $\|\psi(\mathbf{r}, t)\|^2 = \sum_m |\psi_m(\mathbf{r}, t)|^2$. The amplitude of the untrapped wavefunction remains small compared to the condensate wavefunction amplitude by using the assumption of weak coupling [4, 10, 11, 13]. The rf outcoupler is described by the coupling constant:

$$\hbar\Omega_{rf} = g_F \mu_{Bohr} B_{rf} / \sqrt{2}, \quad (5)$$

which denotes to the Ω_{rf} Rabi frequency obtained due to the field B_{rf} for a Lande factor $g_{F=1} = -1/2$. This paper studies hyperfine-manifold $F = 1$ for producing the atom laser, the transition from the magnetically trapped $|1, -1\rangle$ state to the untrapped $|1, 0\rangle$ state. The detuning of two states from the bottom of the trap:

$$\hbar\delta_{rf} = \text{sign}(m)(V_{off} - \hbar\omega_{rf}) \quad (6)$$

where ω_{rf} is the frequency of B_{rf} . Gravity causes a sag in the BEC away from the trap centre at $z = 0$ which depends on M and is given by $z_{sag} = g/|m|\omega_{\perp}^2$. The offset of the trapping potential is denoted by V_{off} for the $m = 0$ condensation in the geometrical center of the trap [4, 11, 14, 15]. In the Thomas-Fermi (TF) approximation, the time dependent ground state of energy μ (the chemical potential) is:

$$\phi_{-1}(\mathbf{r}, t) = \sqrt{\frac{\mu}{U} [1 - \tilde{r}_{\perp}^2 - \tilde{z}^2]}, \quad (7)$$

where $\tilde{r}_{\perp}^2 = (x/x_0)^2 + (y/y_0)^2$, $\tilde{z}^2 = (z/z_0)^2$ and $x_0 = \sqrt{2\mu/M\omega_x^2}$ and $y_0 = z_0 = \sqrt{2\mu/M\omega_{\perp}^2}$, those represent dimensions of the condensation at the frequency of trap $\omega = (\omega_x \omega_{\perp}^2)^{1/3}$. The chemical potential is given by

$$\mu = \frac{\hbar\omega}{2} \left(\frac{15 a N}{\sqrt{\hbar/M\omega}} \right)^{2/5}. \quad (8)$$

At the time dependence of chemical potential μ , each of the condensate ground state of wave function $\Phi(t)$ and the (BEC) dimensions x_0, y_0, z_0 decrease with $N(t)$ [5,14].

Significantly, and upon studying the propagation of the

atom laser, the effective parameters are really quantitative parameters which depend on a limited certain values of other parameters (*i.e.* they depend on the population of atoms in each region). In this paper, the typical values corresponding to the situation of [7, 14] have been taken: $N = 7.2 \times 10^5$ atoms initially, $\omega_x = 2\pi \times 13\text{Hz}$ and $\omega_{\perp} = 2\pi \times 140\text{Hz}$.

3. The wavefunction of Radio Frequency atom laser

The atoms of BEC for $F = 1$ are extracted from the trap by process of rf output-coupling, the rf-outcoupler is described by the energy of detuning $\hbar\delta_{rf}$ and also the term coupling related to the Rabi frequency $\hbar\Omega_{rf}$. The external potential in untrapping coupling system is gravitational potential $V_{ext} = Mgz$. Under the rotating wave approximation in the hyperfine level, the GP equations which express this system are written by formula:

$$i\hbar \frac{\partial \psi_{-1}}{\partial t} = \left[\frac{p^2}{2M} + \hbar\delta_{rf} + V_{trap} + U|\psi_{-1}|^2 \right] \psi_{-1} + \frac{\hbar\Omega_{rf}}{2} \psi_0, \quad (9)$$

$$i\hbar \frac{\partial \psi_0}{\partial t} = \left[\frac{p^2}{2M} + Mgz + U|\psi_{-1}|^2 \right] \psi_0 + \frac{\hbar\Omega_{rf}}{2} \psi_{-1}. \quad (10)$$

To produce output coupling wave function of Radio Frequency atom laser, TF approximation can be used to solve Eq.(10) when dealing with the mean-field term as only a perturbation term compared with gravity. The leading term is therefore the gravitational potential. Therefore, the spatial variations of the mean-field potential can be neglected and consider the following approximated Hamiltonian [14]:

$$H_0 \approx \left[\frac{p^2}{2M} + Mgz \right]. \quad (11)$$

By using Green function for solution, the wave function can be formulated by:

$$\psi_0(\mathbf{r}, t) = \frac{\Omega_{rf}}{2i} \int_0^t dt' \int d^3\mathbf{r}' G_0(\mathbf{r}, t; \mathbf{r}', t') \psi_{-1}(\mathbf{r}', t'), \quad (12)$$

where $(\mathbf{r}, \mathbf{r}')$ represents two different points of the correlation length of the rf atom laser at the time interval $\tau = (t - t')$. The coupled GP equation of untrapping state can be written by Green propagator:

$$\left[i\hbar \frac{\partial}{\partial t} - H_0 \right] G(\mathbf{r}, \mathbf{r}'; \tau) = \delta(\mathbf{r} - \mathbf{r}') \delta(\tau). \quad (13)$$

The Green function can be expressed by the matrix element of operator U_0 between position eigenstates at the time interval $(t - t')$ as [16]:

$$G(\mathbf{r}, \mathbf{r}'; \tau) = \frac{-i}{\hbar} \langle \mathbf{r} | U_0(\tau) | \mathbf{r}' \rangle, \quad (14)$$

where

$$U_0 = \exp[-i H_0(\tau)/\hbar] \quad (15)$$

is a time-evolution operator or propagator. The explicit expression of the propagator G of H_0 can be formulated as:

$$G(\mathbf{r}, \mathbf{r}'; \tau) = C \exp\left[-i \frac{1}{\hbar} \int_{t'}^t H_0 dt\right] \theta(\tau), \quad (16)$$

where $C = \left(\frac{M}{2\pi\hbar\tau}\right)^{3/2}$, $\theta(\tau)$ is the step function. After using the path integrals, the Green function is obtained as follows:

$$G(\mathbf{r}, \mathbf{r}'; \tau) = C \exp[iS_{class}(\mathbf{r}, \mathbf{r}', \tau)] \theta(\tau), \quad (17)$$

where the classical action is given by:

$$S_{class}(\mathbf{r}, \mathbf{r}', \tau) = \pi C^{2/3}[(\mathbf{r} - \mathbf{r}')^2 + g \tau^2 (z + z') - g^2 \tau^4/12]. \quad (18)$$

The output wave function ψ_0 is computed by stationary phase approximations. Therefore, it can obtain the following expression for the outcoupled atomic wavefunction:

$$\psi_0(r, t) = \frac{A(r_{\perp}, z_{res}, t - t_{fall})}{|\zeta_{res}|^{3/2}} \exp\left[\frac{2}{3} i |\zeta_{res}|^{3/2}\right], \quad (19)$$

where

$$A(r_{\perp}, z_{res}, t - t_{fall}) = \frac{\sqrt{\pi} \hbar \Omega_{rf}}{2M g l} \alpha_0(t) \phi_{-1}(r_{\perp}, z_{res}, t) D(t, t_{fall}) \exp[-iE/\hbar], \quad (20)$$

$\alpha_0(t) = \sqrt{(N - N_{-1}(t))/N}$ [6] and E is an energy of system. This equation represents the atom laser mode, $z_{res} = (-\hbar \delta_{rf} + \mu(t))/M g$ is the point of extraction, $t_{fall} = \sqrt{2(z + z_{res})/g}$ is the time of fall from this point, $l = \sqrt[3]{\hbar^2/2M^2 g}$ is a length scale, $\zeta_{res} = \frac{z + z_{res}}{l}$ and $D(t, t_{fall})$ is unity if $0 \leq t_{fall} \leq t$ and zero elsewhere: it describes the finite extent of the atom laser due to the finite coupling time [14]. The density of atom laser propagation is defined through the wavefunction equation by:

$$\rho_0 = |\psi_0|^2 = A(r_{\perp}, z_{res}, t - t_{fall})^2 |\zeta_{res}|^{-1/2}. \quad (21)$$

This expression shows that the density depends on the Rabi frequency, the output coupling period, the detuning frequency and others. Therefore, we study the effect of these parameters on density of atom laser propagation in z direction towards the gravity.

In *Fig.1*, the curves show the change of density versus square Rabi frequency at various values of detuning frequency, output coupling period and the number of atoms in the BEC N . The variable exponential behavior of the density is illustrated in *Fig.1(a)* at the different values of detuning frequency. The unevenness of curves in the figure is subjected to the distribution of atoms in the BEC, the number of atoms in BEC accumulates in the

center and decreases towards the boundaries. From this figure, when square Rabi frequency increases, the number of outgoing atoms N_0 also increases, so density raises exponentially until specific limit against a certain value of Ω_{rf}^2 which is able to extract $N_0 \approx N$. At this point, the occurrence of the negative effect on the density can be observed. In all figures of density, the inset figure shows the small values of density in each figure. *Fig.1(b)* displays the relation of density at $\delta = 0$ kHz (N_0 is the largest) versus square Rabi frequency at different values of the output coupling period. The curves reveal the reverse effect of output coupling period on the density behavior at the values ranging between (10 – 100)ms. This approve the presence of inverse proportionality between output coupling period and square Rabi frequency. From *Fig.1(c)*, it is noted that if the total number of BEC atoms increases, the density behavior decreases. This indicates that the interaction of atoms with each other (the collision) and with the remaining atoms in BEC increases and the dispersion of the outgoing atoms occurs [17].

From *Figs.2*, it is observed that the relationship between density and output coupling period is similar to that of square Rabi frequency except that the density decreasing exceeding those observed in square Rabi frequency curves. in fact, this occurred because the output coupling period affords an opportunity for the outgoing atoms to interact with each other (the collision) and with the remaining atoms in the condensation more than those happened in square Rabi frequency relation with density.

The negative values of detuning in *Figs.3* represent the bottom region of the BEC and the positive values of detuning represent the top region of the BEC while at $\delta = 0$ kHz value represents the region around the center. It is clear that the peak of curves for large values of Rabi frequency is not at $\delta = 0$ kHz, this indicates the asymmetric distribution of atoms in the condensation [7]. The three dimensional diagrams of density at certain values for the wavefunction studied parameters are shown in *Fig.4* at $\delta_{rf} = 0$ kHz and $\delta_{rf} = 10$ kHz . *Table.1* summarizes the peak density when the parameter value is changed in relative to the other parameter values used in the experiments.

Table 1: The peak of density

δ_{rf} (kHz)	0	3	5	8	10
ρ_p ($10^{20} m^{-3}$)	105.11	8.71327	6.1419	2.06283	0.630326
Ω_{rf} (2 π Hz)	200	312	500	700	
ρ_p ($10^{20} m^{-3}$)	15.56	105.11	434.15	11263.8	
t (ms)	10	20	50	100	
ρ_p ($10^{20} m^{-3}$)	46.92	105.11	159.93	1799.55	
N(10^5 atoms)	0.1	1	7.2	50	
ρ_p ($10^{20} m^{-3}$)	5207.24	586.18	105.11	7.3	

4. Conclusion

In this work, the studied system was based on quantitative values of wave function parameters. More precisely, when certain value of the square of Rabi frequency is determined, this value must be proportionate with the energy and the number of atoms localised at a certain extraction point against a certain detuning frequency for the limited period of output coupling.

Our results show that the density behavior of atom laser directly proportional with the square of Rabi frequency and inversely proportional with the period of output coupling, detuning frequency and the total number of BEC. This proportionality depends on the distribution of atoms in the condensate undergoing the Thomas-Fermi distribution. The best region for the best density is the region around the condensate center.

From this research it can be concluded that there is a possibility for controlling the number of released atoms by determining the suitable parameters necessary for producing the best atom laser beam which is mainly used in a many wide applications such as atom optics, atom interferometers and precision measurements.

Disclosures

The authors declare no conflicts of interest.

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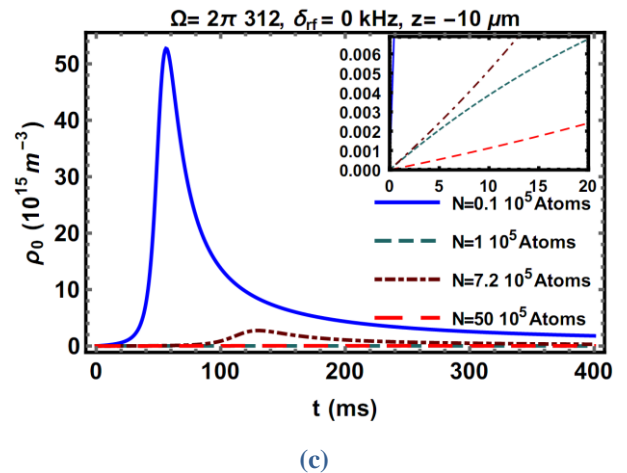
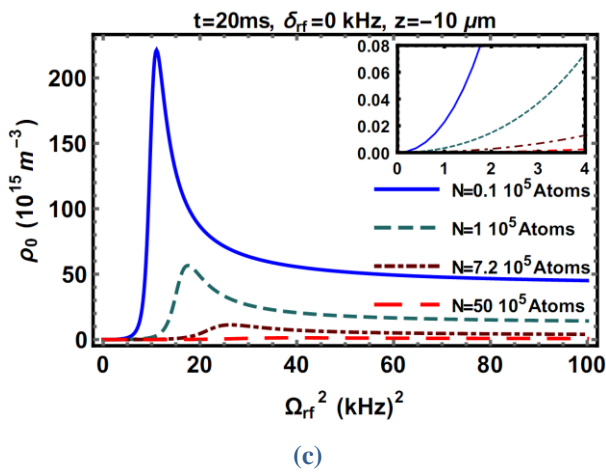
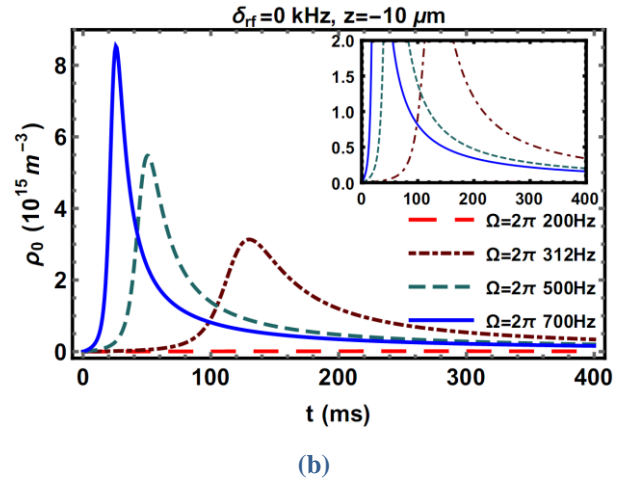
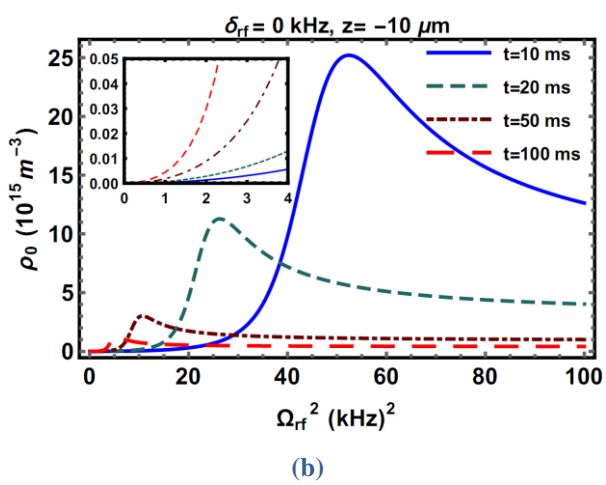
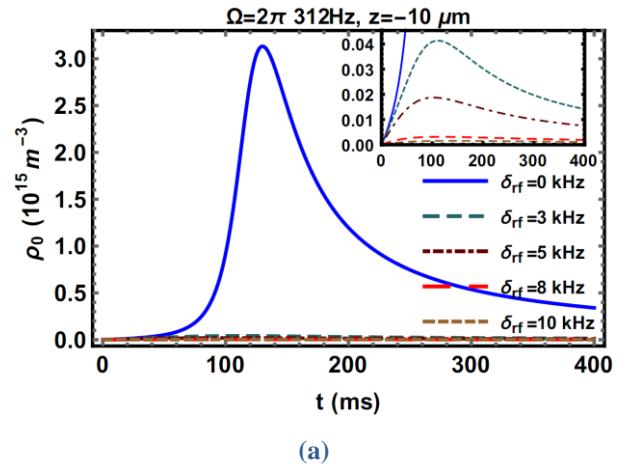
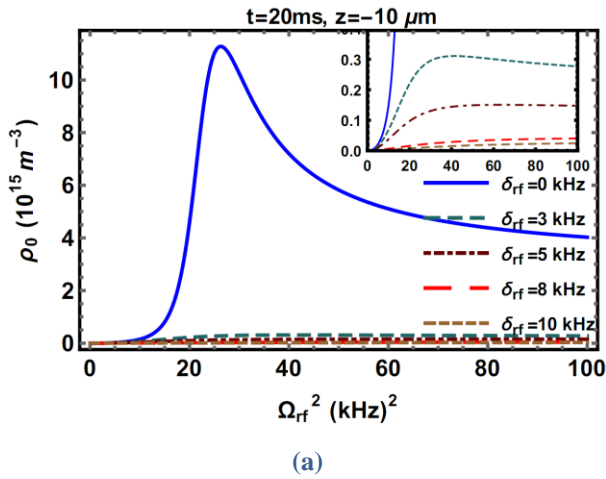
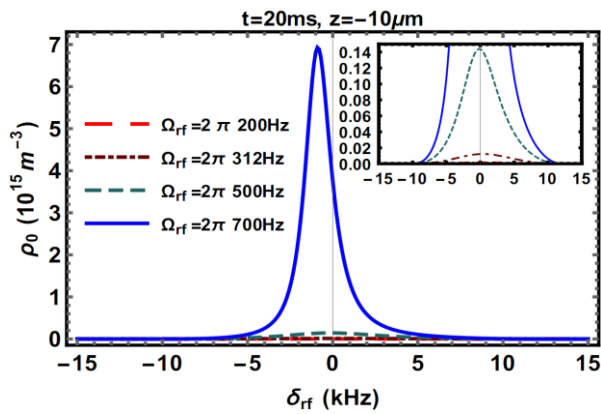
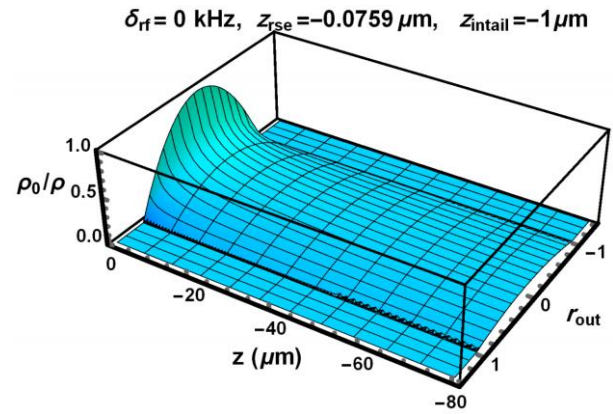


Fig. 1: The effect of the Rabi frequency on the density. (a) At certain values of detuning frequency, (b) at certain values of output coupling period and (c) at certain values of N number of atoms.

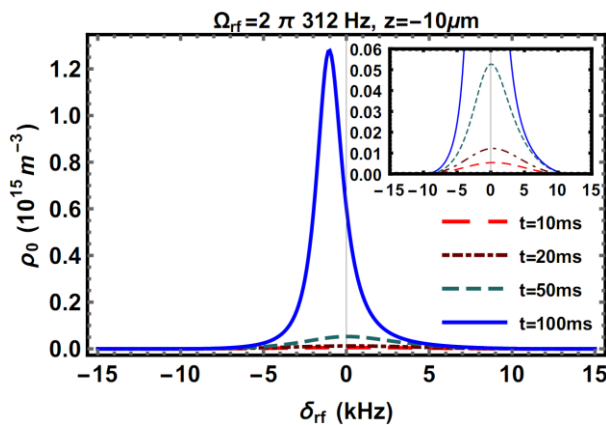
Fig. 2: The effect of the output coupling period on the density. (a) At certain values of detuning frequency, (b) at certain values of Rabi frequency and (c) at certain values of N number of atoms.



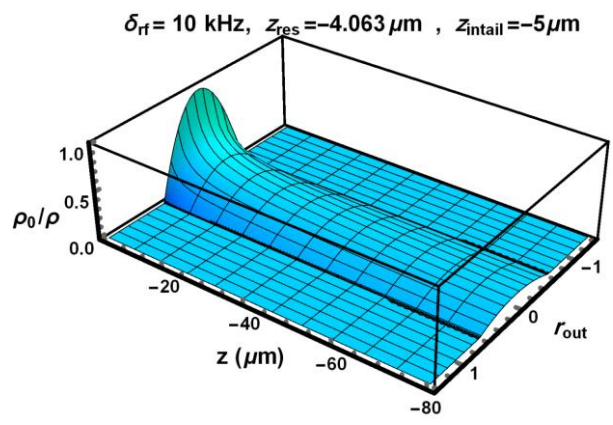
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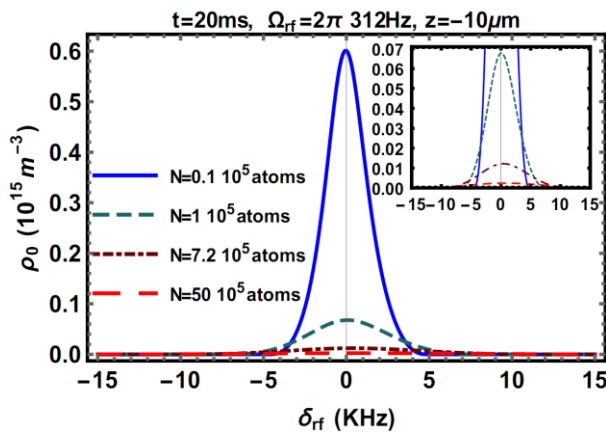
(a)



(b)



(b)



(c)

Fig. 4: 3D diagrams for density at certain values for the wavefunction studied parameters: **(a)** at $\delta = 0\text{kHz}$ and **(b)** $\delta = 10\text{kHz}$.

Fig. 3: The effect of the detuning frequency on the density. **(a)** At certain values of Rabi frequency, **(b)** at certain values of output coupling period and **(c)** at certain values of N number of atoms.

مقالة بحثية

تأثير معاملات دالة الموجة على كثافة الليزر الذرية

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المُلخَص

هذا البحث يدرس تأثير معاملات الدالة الموجية على كثافة الليزر الذري ^{87}Rb . تم اشتقاق معادلة الكثافة من خلال استخدام تقريب توماس فيرمي ودالة قرين وتقريب الطور المستقر. لقد درسنا تغير سلوك الكثافة بتأثير تردد رابي وفترة اقتران الخرج وتردد detuning . وبالتالي يمكن التحكم في عدد الذرات المحررة من خلال تحديد المعلمات المناسبة اللازمة لإنتاج أفضل حزمة ليزر ذري.

الكلمات المفتاحية: تركيز بوز-إنشتاين، الليزر الذري، البصريات الكمية، الموجة المادية.

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