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RESEARCH ARTICLE

EQUALITY OF INERTIAL AND GRAVITATIONAL MASS WITHIN THE FRAMEWORK OF GENERALIZED SPECIAL RELATIVITY AND THE SAVICKAS MODEL

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Abstract

This study examines the equivalence between inertial and gravitational mass within the frameworks of Generalized Special Relativity (GSR) and the Savickas model, with specific objectives to clarify their relationships and implications concerning the principle of equivalence. Our results indicate that, under GSR, inertial mass as measured by an observer in free fall does not equal gravitational mass when both are influenced by gravitational fields. In contrast, the Savickas model consistently supports their equality. This study also highlights that in scenarios involving an accelerating elevator, both models affirm the equality of inertial and gravitational mass, reinforcing the formal definitions of mass in each context. Thus, our findings contribute to a deeper understanding of the implications of mass equivalence in relativistic frameworks. The primary objective is to explore their implications for the equivalence principle and to determine under what conditions these masses remain equal. Our analysis reveals that, in GSR, inertial and gravitational masses are unequal when influenced by gravitational fields, as measured by an external observer. Conversely, the Savickas model consistently supports their equality under similar conditions. Furthermore, in the context of an accelerating elevator, both models affirm the equality of these masses, aligning with classical interpretations of the equivalence principle. These findings provide a deeper understanding of mass equivalence in relativistic frameworks, offering insights into foundational concepts in modern physics.

Keywords: Special relativity, General relativity, Inertial and gravitational mass, Equivalence, Savickas model.

1. Introduction

Einstein's special relativity (SR) and general relativity (GR) theories are among the most significant achievements in physics. Special relativity concerns the concepts of space, time, mass, and energy for observers moving with uniform velocity relative to each other [1]. General relativity addresses gravity and cosmological phenomena [2]. Both theories have been confirmed by experiments to a high degree of precision [3]. However, recently, both SR and GR have faced notable challenges. For instance, the mass generation of elementary particles is not yet well understood within a simple, well-defined model [4, 5]. The Higgs particle, which was claimed to have been discovered, was proposed by some scientists [6]. Unfortunately, mass generation via the Higgs mechanism requires complex mathematics. The effects of fields and the equality of inertial and gravitational masses also present challenges for both SR and GR. Therefore, there is a need for a well-defined, rigorous physical theory that can solve mass-related problems. One of these significant problems is the relationship between inertial and gravitational mass, closely related to the equivalence principle [7]. Some scientists have attempted to link the equality of inertial and gravitational mass to the issues associated with the failures of GR in explaining certain long-standing problems [8]. In this work, generalized special relativity (GSR) [9] and the Savickas expression of mass are utilized to examine whether inertial and gravitational masses are equal. This is discussed in Section 2. Sections 3 and 4 are devoted to discussion and conclusion. This study explores the connection between inertial mass and gravitational mass, focusing on their behavior within the contexts of Generalized Special Relativity (GSR) and the Savickas model [10] The motivation for this comparison stems from the necessity to refine existing models to address gaps in traditional general relativity, particularly in explaining certain phenomena that remain unresolved [11]. GSR attempts to extend the principles of special relativity to account for gravitational influences, integrating them into a more intricate spacetime framework [12]. On the other hand, the Savickas model offers a distinct viewpoint that corresponds with various experimental observations regarding the equivalence of mass types [13]. By examining these two approaches, we aim to uncover their core differences and assess how each contributes to the broader interpretation of the equivalence principle within relativistic theory. [14], We revise the concept of mass of a particle in general relativity initiated by Einstein, Brans, and Rosen in the fifties, using the results of P. Havas and J.N. Goldberg's analysis of the motion equations for point particles. We show how one can define a constant inertial mass and a variable gravitational mass dependent on their gravitational interaction with the rest of the particles. The concept of gravitational mass enables the development of a cosmological model that effectively explains the observed mass deficit, dark energy, and the cosmological constant, without requiring the introduction of new types of matter or energy. In this model, dark matter and dark energy are interpreted as gravitational phenomena within the framework of standard general relativity. We have applied the findings of P. Havas and J.N. Goldberg regarding the dynamics of a finite set of gravitating point particles to revisit the definitions of inertial and gravitational mass. Portilla, Miguel [15], The comparison between Generalized Special Relativity (GSR) and Savickas' model arises from the persistent challenge of clarifying the relationship between inertial and gravitational mass, a fundamental aspect of modern physics connected to Einstein's equivalence principle. This principle, central to General Relativity (GR), asserts that the effects of gravity are locally indistinguishable from those of acceleration. [16] While GR has been highly successful in describing a wide range of physical phenomena [17], it faces challenges in addressing unresolved issues like dark matter, dark energy, and the integration of gravity with quantum mechanics. GSR builds on special relativity by integrating gravitational fields into its spacetime framework [18]. By doing so, it introduces a dependency of mass on gravitational potential and field effects, providing a new perspective on how gravitational interactions affect the equivalence of inertial and gravitational mass. In contrast, Savickas' model presents a straightforward approach that consistently supports the equality of inertial and gravitational mass across various scenarios. It aligns well with experimental observations, offering a robust interpretation of mass equivalence even under the influence of gravitational effects. The motivation for comparing these models lies in the opportunity to address conceptual uncertainties about the nature and behavior of mass within relativistic theories. Analyzing their similarities and differences provides valuable insights into the equivalence principle and its role in systems subject to gravitational or accelerating forces. This understanding is essential for advancing theoretical physics and bridging the gaps between established frameworks [19,20].

2. Inertial and Gravitational Mass

The dependence of particle masses on fields and motion was addressed by many scientists [21,22]. Before Einstein's SR theory, particle masses were considered universal constants. However, special relativity demonstrates that the mass mmm increases according to the relation [23]

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1}$$

Where m_0 is the rest mass, thus The mass is affected by the potential and the field, as expressed in the following relation [24]

$$m = \frac{g_{00} m_0}{\sqrt{g_{00} - \frac{v^2}{c^2}}} \tag{2}$$

Where g_{00} is given by [25].

$$g_{00} = 1 + \frac{2\phi}{c^2} \tag{3}$$

This model is called Generalized Special Relativity (GSR). The dependence of mass on the field in curved spacetime has also been proposed by Savickas as follows [26]:

$$m = \frac{m_0}{\sqrt{g_{00} - \frac{v^2}{c^2}}} \tag{4}$$

In his general relativity theory, Einstein proposed the equality of inertial mass m_i and gravitational mass m_g according to the so-called equivalence principle, one can see the compatibility of this principle with GSR and the Savickas model [27].

consider now a particle at rest in an elevator falling freely with the particle, From the perspective of an observer on Earth, the speed of the particle is

$$v^2 = v_0^2 + 2ax = 2ax = 2\phi \tag{5}$$

He sees the particle moving with speed v in the field (ϕ). Thus, according to him, equations (2), (3), and (5), the gravitational mass is given by

$$m_g = \frac{\left(1+\frac{2\phi}{c^2}\right)m_0}{\sqrt{1+\left(\frac{2\phi - v^2}{c^2}\right)}} = \left(1+\frac{2\phi}{c^2}\right)^{\frac{1}{2}} m_0 \qquad (6)$$

Where gravitational mass m_g is defined as a measure of an object's response to gravitational force or its role as a source of gravitational interaction, according to Newton's Law of Universal Gravitation

$$F = G\frac{m_g M}{r^2}.$$

Equation (6) above describes the gravitational mass $m_g$in the context of the GSR model, where the mass varies depending on the gravitational potential ∅ and velocity v. This equation It demonstrates how the gravitational mass varies in response to both the gravitational potential and the particle's velocity. In the case of the elevator, the particle remains stationary, and no acceleration is detected. Therefore,

$$v = 0 \quad a = 0 \quad ax = \phi = 0 \qquad (7)$$

Hence, equation (2) reads

$$m_i = \frac{m_0}{\sqrt{1}} = m_0 \qquad (8)$$

Inertial Mass m_i : Defined as a measure of an object's resistance to acceleration when a force is applied, as used in Newton's Second Law $F = ma_i$.

Thus, according to the GSR model

$$m_i \neq m_g \qquad (9)$$

i.e., the inertial mass is not equal to the gravitational mass. In contrast, within the framework of the Savickas model, equations (5) and (6) yield the following results for an observer on Earth [28].

$$m_g = \frac{m_0}{\sqrt{1+\left(\frac{2\phi - v^2}{c^2}\right)}} = m_0 \qquad (10)$$

Equation (10) above describes the gravitational mass in the model. This equation suggests that the gravitational mass remains constant ($m_g = m_0$) irrespective of the velocity or the gravitational potential. In contrast to the GSR model, the Savickas model maintains the equality between inertial and gravitational masses in all situations, aligning with the equivalence principle.

While for the elevator observer

$$m_i = m_0 \qquad (11)$$

Thus, the inertial and gravitational masses are equal. One can tackle the problem in another way by considering a particle falling freely in gravity and another one an elevator moving in free space with acceleration g with respect to a particle of mass m_0 .the elevator accelerating in free space refers to a hypothetical setup where the elevator, under a constant force, accelerates in the absence of any external gravitational field. This acceleration is assumed to be uniform and constant, similar to the acceleration that would occur in a gravitational field (as per the equivalence principle), but it is crucial to note that the force causing the elevator's motion is not a gravitational force but rather an external propulsion force.

For the particle moving in gravity

$$v^2 = 2ax = 2\phi \qquad \phi \neq 0 \qquad (12)$$

The mass expression in GSR reads[30].

$$m_g = \frac{\left(1+\frac{2\phi}{c^2}\right)m_0}{\sqrt{1+\left(\frac{2\phi - v^2}{c^2}\right)}} = \left(1+\frac{2\phi}{c^2}\right)^{\frac{1}{2}} m_0 \qquad (13)$$

While for an elevator in free space, the same equation gives

$$v^2 = 2gx = 2\phi \qquad (14)$$

$$m_i = \frac{\left(1+\frac{2\phi}{c^2}\right)m_0}{\sqrt{1+\left(\frac{2\phi - v^2}{c^2}\right)}} = \left(1+\frac{2\phi}{c^2}\right)^{\frac{1}{2}} m_0 \qquad (15)$$

According to Generalized Special Relativity (GSR), the gravitational mass and inertial mass are identical. Fortunately, the Savickas model for Earth [see (4) and (12)] gives

$$m = \frac{m_0}{\sqrt{1+\left(\frac{2\phi - v^2}{c^2}\right)}} = m_0 \qquad (16)$$

While for elevator [see (2) and (14)]

$$m_i = \frac{m_0}{\sqrt{1-\left(\frac{2\phi - v^2}{c^2}\right)}} = \frac{m_0}{\sqrt{1}} = m_0 \qquad (17)$$

Again, the gravity and inertial mass are equal in the Savickas model.

$$m_g = m_i \qquad (18)$$

In relativistic physics, the relationship between mass and velocity becomes more complex as objects approach significant fractions of the speed of light. The figures below illustrate this behavior in the context of Generalized Special Relativity (GSR) and the Savickas model.

Table 1: Comparison Between Inertial Mass and Gravitational Mass Across Different Reference Frames and Models

	Inertial Mass (m_i)	**Gravitational Mass (m_g)**
Definition	Resistance of a body to acceleration when a force is applied $F = m_i a$	Measure of a body's response to gravitational force or its ability to generate gravitational attraction $F = G\frac{m_g M}{r^2}$
Reference Frame: Accelerating Elevator	Equivalent to rest mass (m_0) because no relative acceleration is observed($m_i = m_0$)	Equivalent to rest mass ($m_g = m_0$) as gravitational effects are not measurable in this frame.
Reference Frame: Gravitational Field	Dependent on velocity and gravitational potential as per the GSR model ($m_i \neq m_0$)	Varies with gravitational potential and velocity $m_g = \frac{\left(1+\frac{2\emptyset}{c^2}\right)m_0}{\sqrt{1+\frac{(2\emptyset - v^2)}{c^2}}}$
GSR Model	Mass changes due to both motion (v) and gravitational potential ($\emptyset$)	Gravitational effects alter mass, leading to inequality between m_i and m_g n some cases.
Savickas Model	Maintains equality with gravitational mass regardless of velocity or potential ($m_i = m_g$)	Remains constant across different scenarios, aligning with the equivalence principle ($m_i = m_g$)
present Model	Demonstrates the principle holds in some models but not in GSR for external observers in gravitational fields.	Fully aligns with equivalence principle in Savickas model, supporting mass equality.

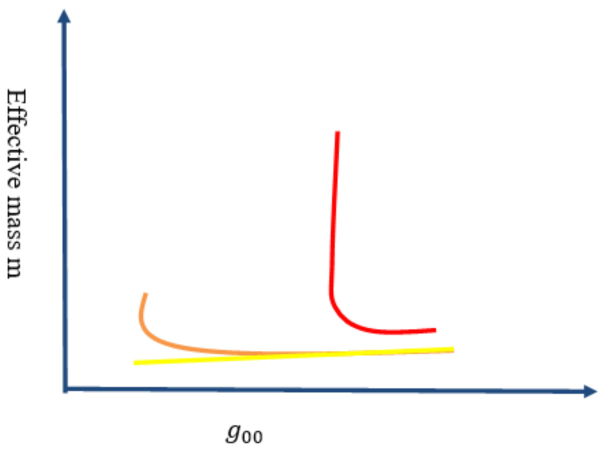

Fig. 1: Theoretical relation of the effective mass m and the gravitational component g_{00} based on Equation (2.2). The plot displays the effective mass for different velocity

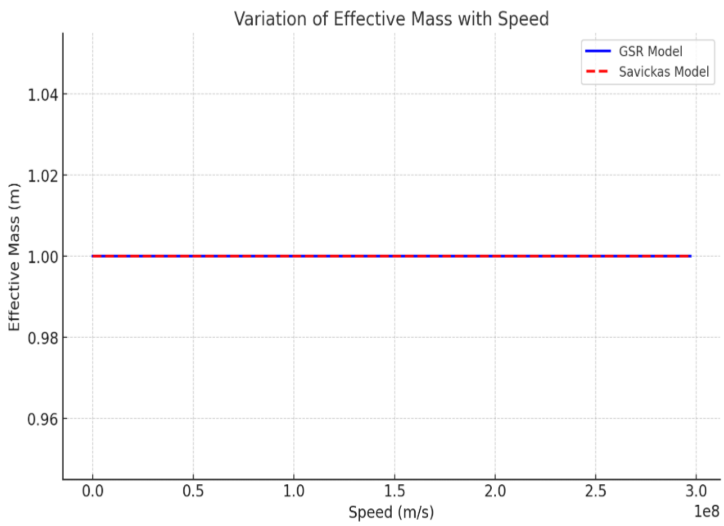

Fig. 2: Theoretical relation of the change in mass with speed for GSR and Savickas model

The effective mass m changes with the gravitational component g_{00} at different velocities. At lower velocities, the mass primarily depends on g_{00}, increasing as g_{00} decreases, demonstrating the influence of gravity on mass. As the velocity increases, the mass becomes more sensitive to changes in g_{00}, with relativistic effects amplifying the impact of gravity. At higher velocities, even small variations in g_{00} lead to significant changes in the effective mass, illustrating the combined effects of high speed and gravity. The relationship between mass and velocity transitions from a linear to a nonlinear pattern as speed increases, highlighting the interaction between speed and gravitational effects on mass, the figure confirms the equation (2.2) and helps us understand mass changes in strong gravity and high-speed conditions.

Fig (2) above shows how that the plots comparing the change in mass with speed for both the Generalized Special Relativity (GSR) and Savickas models. The plot shows how the effective mass mmm changes with speed v, for different values of the gravitational potential parameter g_{00}. The differences in mass behavior can be observed as the speed approaches a significant fraction of the speed of light. The curves for both models are shown with solid lines for the GSR model and dashed lines for the Savickas model

3. Discussion

The inertial mass can be defined as the mass of a particle in unaccelerated inertial frame, while gravitational mass is the mass of a particle falling with respect to an observer in gravity field.

According to this definition GSR shows that the inertial and gravitational mass are not equal according to equations (6), (8) and (9). However, Savickas model shows that they are equal according to equations (10) and (11). The inertial mass can in contray defined as the mass of a particle moving in free space with acceleration with respect to an observer, In view of this definition , both inertial mass and gravitational mass are equal according to GSR and Savickas model, Our results confirm that the Generalized Special Relativity (GSR) model differs from the Savickas model in the relationship between inertial mass and gravity, where the masses remain equal in the Savickas model under all gravitational conditions, while the GSR model emphasizes a disparity between them in the presence of gravitational fields.

Future research should focus on studying the effects of gravitational fields in additional models that may integrate general relativity and special relativity more deeply, contributing to solving unresolved phenomena such as dark matter and dark energy.

4. Conclusion

The equality of inertial mass and gravitational mass depends on the definition of the inertial mass with in the frame work of GSR. But in Savickas model the equality does not affect by the definition of inertial mass, Our results confirm that the Generalized Special Relativity (GSR) model differs from the Savickas model in the relationship between inertial mass and gravity. In the Savickas model, the masses remain equal under all gravitational conditions, whereas the GSR model indicates a disparity between them in the presence of gravitational fields, Future research should focus on studying the effects of gravitational fields in additional models that could more deeply integrate General and Special Relativity. This could help address unresolved phenomena, such as dark matter and dark energy.

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مقالة بحثية

تساوي الكتلة القصورية وكتلة الجاذبية في إطار النسبية الخاصة المعممة ونموذج سافيكاس

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المُلخّص

تتناول هذه الدراسة التماثل بين الكتلة القصورية مع الكتلة الجاذبية ضمن أطر النسبية الخاصة المعممة (GSR) ونموذج سافيكاس، مع أهداف محددة لتوضيح علاقاتهما وآثارهما المتعلقة بمبدأ التكافؤ. تشير النتائج إلى أنه، في إطار النسبية الخاصة المعممة، لا تتساوى الكتلة القصورية كما يقيسها مراقب في حالة السقوط الحر مع الكتلة الجاذبية عندما يتأثر كلاهما بالحقول الجاذبية. بالمقابل، يدعم نموذج سافيكاس التماثل بين الكتلتين بشكل مستمر. تبرز هذه الدراسة أيضًا أنه في الحالات التي تتضمن مصعدًا متسارعًا، يؤكد كل من النموذجين على تكافؤ الكتلة القصورية والجاذبية، مما يعزز التعريفات الرسمية للكتلة في كل سياق. وبالتالي، تساهم نتائج هذه الدراسة في فهم أعمق لآثار تماثل الكتل في الأطر النسبية. الهدف الأساسي هو استكشاف آثار هذه العلاقة على مبدأ التكافؤ وتحديد الظروف التي تظل فيها الكتل متساوية. تكشف تحليلات هذه الدراسة أن الكتلتين القصورية والجاذبية غير متساويتين في إطار النسبية الخاصة المعممة عندما يتأثران بالحقول الجاذبية، كما يقيسهما مراقب خارجي. في المقابل، يدعم نموذج سافيكاس باستمرار تكافؤهما تحت نفس الظروف. علاوة على ذلك، في سياق المصعد المتسارع، يؤكد النموذجان تكافؤ هاتين الكتلتين، بما يتماشى مع التفسيرات الكلاسيكية لمبدأ التكافؤ. تقدم هذه النتائج فهماً أعمق لتماثل الكتل في الأطر النسبية مع تقديم رؤى اعمق حول المفاهيم الأساسية في الفيزياء الحديثة.

الكلمات المفتاحية: النسبية الخاصة، النسبية العامة، الكتلة القصورية والكتلة الجاذبية، التكافؤ، نموذج سافيكاس.

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