Electronic Journal of University of Aden for Basic and Applied Sciences
EJUA-BA Vol. 7 No. 1 (2026)
https://doi.org/10.47372/ejua-ba.2026.1.494
ISSN: 2708-0684

RESEARCH ARTICLE

ON (*K, P, S*)-GENERALIZATIONS OF THE GAMMA, BETA AND HYPERGEOMETRIC FUNCTIONS AND THEIR PROPERTIES

Maisoon Ahmed Kulaib[1,*], and Ahmed Ali Atash[2]

[1] *Dept. of Basic Sciences (Mathematics), Faculty of Engineering, University of Aden, Aden, Yemen*
[2] *Dept. of Mathematics, Faculty of Education, Shabwah University, Shabwah, Yemen*

***Corresponding author: Maisoon Ahmed Kulib; E-mail: maisoonahmedkulib@gmail.com**

Received: 26 January 2026 / Accepted: 09 February 2026 / Published online: 31 March 2026

Abstract

In this paper, we introduce a unified three-parameter ((*k,p,s*))-generalization of the Pochhammer symbol, Gamma and Beta functions. Based on these definitions, a corresponding ((*k,p,s*))-generalized hypergeometric function is defined. Several fundamental properties are derived, including functional equations, summation formulas and integral representations. It is shown that many known extensions of special functions arise as particular cases of the proposed framework, thereby unifying and extending earlier results in the literature.

Keywords: (*k-p-s*)-Pochhammer's symbol; (*k-p-s*)-Gamma function; (*k-p-s*)- beta functions; (*k-p-s*)-generalized hypergeometric function; Integral representations.
MSC 2010: 33B15, 33C05, 33C20.

1. Introduction

Special functions are naturally generalizations of the elementary functions and they play a vital role in the solution of differential equations. These functions have also a wide application in many branches of mathematics, physics and engineering. Recently, various extensions and properties of Gamma and Beta functions have been introduced from time to time by several authors (see [1], [2], [3], [4], [5], [6],[7]). Also, in many recent works, the extended Gamma and Beta functions and their generalizations are used to define a new extension of some special functions such as Gauss's, Appell's, Lauricella's, Srivastava's and Exton's hypergeometric functions (see [8], [9], [10], [11], [12], [13]). In 2007, Diaz and Pariguan [14] have introduced k-Pochhammer's symbol, *k*-Gamma and *k*-Beta functions as follows:

$$(a)_{n,k} = a(a+k)(a+2k)\cdots(a+(n-1)k), \quad n \in N, k > 0, \tag{1.1}$$

$$\Gamma_k(x) = \int_0^\infty t^{x-1} \exp\left(-\frac{t^k}{k}\right) dt, \quad Re(x) > 0, k > 0 \tag{1.2}$$

and

$$B_k(x,y) = \frac{1}{k}\int_0^1 t^{\frac{x}{k}-1}(1-t)^{\frac{y}{k}-1} dt, \quad Re(x) > 0, Re(y) > 0. \tag{1.3}$$

Remark .1.1. If we put $k = 1$, in the equations (1.1), (1.2) and (1.3), then we get the classical Pochhammer's symbol, Gamma and Beta functions [15]

$$(a)_n = a(a+1)(a+2)\cdots(a+(n-1)), \quad a \in C, n \in N, \tag{1.4}$$

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\, dt, Re(x) > 0 \tag{1.5}$$

and

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\, dt, (Re(x) > 0, Re(y) > 0). \tag{1.6}$$

Gehlot [16] have introduced another form of extended Pochhammer's symbol, Gamma and Beta functions of two parmeter as follows:

$$(a)_{n,k}^p = \left(\frac{ap}{k}\right)\left(\frac{ap}{k}+p\right)\left(\frac{ap}{k}+2p\right)\ldots\left(\frac{ap}{k}+(n-1)p\right),$$

$$a \in C, n \in N, k, p > 0, \qquad (1.7)$$

$${}_p\Gamma_k(x) = \int_0^\infty t^{x-1} \exp\left(\frac{-t^k}{p}\right) dt,$$

$$Re(x) > 0, k, p > 0 \qquad (1.8)$$

and

$${}_pB_k(x,y) = \frac{1}{k}\int_0^1 t^{\frac{x}{k}-1}(1-t)^{\frac{y}{k}-1} dt,$$

$$Re(x) > 0, Re(y) > 0. \qquad (1.9)$$

Remark.1.2. If we put $k = p = 1$ in the equations (1.7), (1.8) and (1.9), then we get the classical Pochhammer's symbol, Gamma and Beta functions given in (1.4), (1.5) and (1.6).

Recently, Mubeen et al. [17] have introduced a new generalization of Pochhammer's symbol, Gamma and Beta functions as follows:

$${}^p(a)_n^s = a\left(a+\frac{p}{s}\right)\ldots\left(a+(n-1)\frac{p}{s}\right),$$

$$a \in C, n \in N, p, s > 0, \qquad (1.10)$$

$${}^p\Gamma^s(x) = \int_0^\infty t^{x-1} \exp\left(-\frac{st^{\frac{p}{s}}}{p}\right) dt, \qquad (1.11)$$

$$x \in \mathbb{C}, Re(x) > 0, p, s \in \mathbb{R}^+$$

and

$${}^pB^s(x,y) = \frac{s}{p}\int_0^1 t^{\frac{sx}{p}-1}(1-t)^{\frac{sy}{p}-1} dt, \qquad (1.12)$$

$$Re(x) > 0, Re(y) > 0, p, s \in \mathbb{R}^+.$$

Remark.1.3. If we put $p = s = 1$, in the equations (1.10), (1.11) and (1.12), then we get the classical Pochhammer's symbol, Gamma and Beta functions given in (1.4), (1.5) and (1.6).

Motivated by these developments, the main objective of this paper is to introduce a unified three-parameter extension that simultaneously generalizes the k-functions of Diaz and Pariguan, the (p,k)-functions of Gehlot and the (p,s)-functions of Mubeen et al. The proposed framework provides a systematic approach to derive known results as special cases while yielding new identities and integral representations.

2. *(k-p-s)*- Pochhammer's symbol

In this section, we define a new generalization of Pochhammer's symbol as follows:

Definition 2.1. The three-parameter generalized Pochhammer's symbol is defined as:

$$(a)_{n,k}^{p,s} = \prod_{j=0}^{n-1}\left(\frac{jp}{s}+\frac{ap}{sk}\right), \qquad (2.1)$$

for $a \in \mathbb{C}; p, k, s \in \mathbb{R}^+; n \in N$.

Particular cases:

i) For $s = 1, p = k$, equation (2.1) reduces to k-Pochhammer's symbol defined by Diaz and pariguan [14]

$$(a)_{n,k}^{k,1} = (a)_{n,k}.$$

ii) For $s = 1$, equation (2.1) reduces to p- k Pochhammer's symbol defined by Gehlot [16]

$$(a)_{n,k}^{p,1} = {}_p(a)_{n,k}.$$

iii) For $k = \frac{p}{s}$, equation (2.1) reduces to s-p Pochhammer's symbol defined by Mubeen et al. [17]

$$(a)_{n,\frac{p}{s}}^{p,s} = {}^p(a)_n^s.$$

iv) For $s = p = k = 1$, equation (2.1) reduces to Pochhammer's symbol [15]

$$(a)_{n,1}^{1,1} = (a)_n.$$

Theorem 2.1.

$$(a)_{n,k}^{p,s} = \left(\frac{p}{s}\right)^n \left(\frac{a}{k}\right)_n, \qquad (2.2)$$

$$(a)_{mn,k}^{p,s} = \left(\frac{pm}{s}\right)^{mn} \left(\frac{\frac{a}{k}}{m}\right)_n \left(\frac{\left(\frac{a}{k}\right)+1}{m}\right)_n \left(\frac{\left(\frac{a}{k}\right)+2}{m}\right)_n \ldots \left(\frac{\left(\frac{a}{k}\right)+m-1}{m}\right)_n, \qquad (2.3)$$

$$(a)_{m+n,k}^{p,s} = (a)_{m,k}^{p,s}\,(a+mk)_{n,k}^{p,s}, \qquad (2.4)$$

$$(a)_{m-n,k}^{p,s} = \frac{(-1)^n (a)_{m,k}^{p,s}}{((1-m)k-a)_{n,k}^{p,s}}. \qquad (2.5)$$

Proof of (2.2). From (2.1), we have

$$(a)_{n,k}^{p,s} = \left(\frac{ap}{sk}\right)\left(\frac{ap}{sk}+\frac{p}{s}\right)\left(\frac{ap}{sk}+\frac{2p}{s}\right)\ldots\left(\frac{ap}{sk}+(n-1)\frac{p}{s}\right)$$
$$= \left(\frac{p}{s}\right)^n\left(\frac{a}{k}\right)\left(\frac{a}{k}+1\right)\left(\frac{a}{k}+2\right)\ldots\left(\frac{a}{k}+(n-1)\right),$$

which by using definition (1.4), we get the desired result.

Proof of (2.3). From (2.2), we have

$$(a)_{mn,k}^{p,s} = \left(\frac{p}{s}\right)^{mn}\left(\frac{a}{k}\right)_{mn}.$$

Using the multiplication formula

$$(a)_{mn} = m^{mn}\left(\frac{a}{m}\right)_n\left(\frac{a+1}{m}\right)_n \ldots \left(\frac{a+m-1}{m}\right)_n,$$

we obtain the desired result.

Proof of (2.4). From (2.2), we have

$$(a)_{m+n,k}^{p,s} = \left(\frac{p}{s}\right)^{m+n}\left(\frac{a}{k}\right)_{m+n}$$

$$= \left(\frac{p}{s}\right)^{n+m} \left(\frac{a}{k}\right)_m \left(\frac{a}{k}+m\right)_n$$

$$= (a)_{m,k}^{p,s} \left(\frac{p}{s}\right)^n \left(\frac{a}{k}+m\right)_n$$

$$= (a)_{m,k}^{p,s} \left(\frac{p}{s}\right)^n \left(\frac{a}{k}+m\right)\left(\frac{a}{k}+m+1\right)\dots\left(\frac{a}{k}+m + (n-1)\right)$$

$$= (a)_{m,k}^{p,s} \left(\frac{p(a+mk)}{sk}\right)\left(\frac{p(a+mk)}{sk}+\frac{p}{s}\right) \times \left(\frac{p(a+mk)}{sk}+\frac{2p}{s}\right)\dots\left(\frac{p(a+mk)}{sk}+(n-1)\frac{p}{s}\right),$$

which by using definition (2.1), we get the desired result.

Proof of (2.5). From (2.2), we have

$$(a)_{m-n,k}^{p,s} = \left(\frac{p}{s}\right)^{m-n} \left(\frac{a}{k}\right)_{m-n}.$$

Using the following identity:

$$(a)_{m-n} = \frac{(-1)^n (a)_m}{(1-a-m)_n}$$

and (2.2), we obtain the desired result.

Theorem 2.2.

$$\sum_{n=0}^{\infty} \frac{(a)_{n,k}^{p.s}}{n!} t^n = \left(1-\frac{pt}{s}\right)^{\frac{-a}{k}}, \left(\left|{}^{pt}/{}_{s}\right| < 1\right), \quad (2.6)$$

$$\frac{\partial}{\partial p}(a)_{n,k}^{p,s} = \frac{n}{p}(a)_{n,k}^{p,s}, \quad (2.7)$$

$$\frac{\partial}{\partial s}(a)_{n,k}^{p,s} = -\frac{n}{s}(a)_{n,k}^{p,s}. \quad (2.8)$$

Proof of (2.6). Using (2.2) and the binomial theorem, we obtain the desired result.

Proof of (2.7) and (2.8). The derivatives are obtained after expressing the symbol in terms of the classical Pochhammer symbol as given in (2.2), we obtain the desired result.

3. *(k-p-s)*- Gamma function

In this section, we define a new generalization of Gamma function as follows:

Definition 3.1. The three-parameter generalized Gamma function is defined as:

$${}_p\Gamma_k^s(x) = \int_0^\infty t^{x-1} \exp\left(-\frac{st^k}{p}\right) dt, \quad (3.1)$$

For $x \in \mathbb{C}, Re(x) > 0\,; p, k, s \in \mathbb{R}^+$.

Particular cases:

i) For $s = 1, p = k$, equation (3.1) reduces to *k*- Gamma function defined by Diaz and pariguan [14]

$${}_k\Gamma_k^1(x) = \Gamma_k(x).$$

ii) For $s = 1$, equation (3.1) reduces to *p*- *k* Gamma function defined by Gehlot [16]

$${}_p\Gamma_k^1(x) = {}_p\Gamma_k(x).$$

iii) For $k = \frac{p}{s}$, equation (3.1) reduces to *s*-*p* Gamma function defined by Mubeen et al. [17]

$${}_p\Gamma_{\frac{p}{s}}^s(x) = {}^s\Gamma^p(x).$$

iv) For $s = p = k = 1$, equation (3.1) reduces to Gamma function [15]

$${}_1\Gamma_1^1(x) = \Gamma(x).$$

Theorem 3.1.

$${}_p\Gamma_k^s(x) = \frac{1}{k}\left(\frac{p}{s}\right)^{\frac{x}{k}} \Gamma\left(\frac{x}{k}\right), \quad (3.2)$$

$${}_p\Gamma_k^s(x+k) = \frac{px}{ks}\ {}_p\Gamma_k^s(x), \quad (3.3)$$

$$(a)_{n,k}^{s,p} = \frac{{}_p\Gamma_k^s(a+nk)}{{}_p\Gamma_k^s(a)}. \quad (3.4)$$

Proof of (3.2). Using the substituting $u = \frac{s}{p}t^k$ in (3.1), we find

$${}_p\Gamma_k^s(x) = \frac{1}{k}\left(\frac{p}{s}\right)^{\frac{x}{k}} \int_0^\infty u^{\frac{x}{k}-1} e^{-u} du.$$

Using (1.5) in the above equation, we obtain the desired result.

Proof of (3.3). Using (3.2), we have

$${}_p\Gamma_k^s(x+k) = \frac{1}{k}\left(\frac{p}{s}\right)^{\frac{x}{k}+1} \Gamma\left(\frac{x}{k}+1\right)$$

$$= \frac{px}{ks}\left\{\frac{1}{k}\left(\frac{p}{s}\right)^{\frac{x}{k}} \Gamma\left(\frac{x}{k}\right)\right\},$$

which by using (3.2), we get the desired result.

Proof of (3.4). Equation (3.4) generalizes the classical relation between the Pochhammer symbol and the Gamma function. Denote the right-hand side of (3.4) by R and using (3.2), we have

$$R = \frac{\left(\frac{p}{s}\right)^n \Gamma\left(\frac{a}{k}+n\right)}{\Gamma\left(\frac{a}{k}\right)}$$

$$R = \left(\frac{p}{s}\right)^n \left(\frac{a}{k}\right)_n,$$

which by using definition (2.2), we obtain the Left-hand side of (3.4). This completes the proof of **Theorem 3.1.**

4. *(k-p-s)*- Beta function

In this section, we define a new generalization of Beta function as follows:

Definition 4.1. The three-parameter Beta function is defined as:

$$ {}_pB_k^s(x,y) = \frac{{}_p\Gamma_k^s(x)\,{}_p\Gamma_k^s(y)}{{}_p\Gamma_k^s(x+y)}, \tag{4.1}$$

$$Re(x) > 0, Re(y) > 0, k,p,s \in \mathbb{R}^+.$$

Particular cases:

i) For $s = 1, p = k$, equation (3.1) reduces to k- Beta function defined by Diaz and pariguan [14]

$${}_kB_k^1(x,y) = B_k(x,y).$$

ii) For $s = 1$, equation (3.1) reduces to p- k- Beta function defined by Gehlot [16]

$${}_pB_k^1(x,y) = {}_pB_k(x,y).$$

iii) For $k = \frac{p}{s}$, equation (3.1) reduces to s-p- Beta function defined by Mubeen et al. [17]

$${}_pB_{\frac{p}{s}}^s(x,y) = {}^pB^s(x,y).$$

iv) For $s = p = k = 1$, equation (3.1) reduces to Beta function [15]

$${}_1B_1^1(x,y) = B(x,y).$$

Theorem 4.1.

$${}_pB_k^s(x,y) = \frac{1}{k}\int_0^1 t^{\frac{x}{k}-1}(1-t)^{\frac{y}{k}-1}dt, \tag{4.2}$$

$$\frac{(\beta)_{n,k}^{p,s}}{(\gamma)_{n,k}^{p,s}} = \frac{{}_pB_k^s(\beta+nk,\gamma-\beta)}{{}_pB_k^s(\beta,\gamma-\beta)}. \tag{4.3}$$

Proof of (4.2). Employing (3.2) into (4.1), we get the desired result.

Proof of (4.3). Consider the left-hand side of (4.3) and using (3.4), we have

$$\frac{(\beta)_{n,k}^{p,s}}{(\gamma)_{n,k}^{p,s}} = \frac{{}_p\Gamma_k^s(\beta+nk)}{{}_p\Gamma_k^s(\beta)}\,\frac{{}_p\Gamma_k^s(\gamma)}{{}_p\Gamma_k^s(\gamma+nk)}$$
$$= \frac{{}_p\Gamma_k^s(\beta+nk)\,{}_p\Gamma_k^s(\gamma-\beta)}{{}_p\Gamma_k^s(\gamma+nk)}\,\frac{{}_p\Gamma_k^s(\gamma)}{{}_p\Gamma_k^s(\beta)\,{}_p\Gamma_k^s(\gamma-\beta)},$$

which by using (4.1), we get the desired result.

5. *(k-p-s)*- Generalized hypergeometric function

Definition 5.1. The three-parameter generalized hypergeometric function is defined as:

$${}_{q+1}F_q^{k,p,s}(\alpha,\beta_1,\dots,\beta_q;\gamma_1,\dots,\gamma_q;x)$$

$$= \sum_{n=0}^{\infty}\frac{(\alpha)_{n,k}^{p,s}(\beta_1)_{n,k}^{p,s}\dots(\beta_q)_{n,k}^{p,s}}{(\gamma_1)_{n,k}^{p,s}\dots(\gamma_q)_{n,k}^{p,s}}\frac{x^n}{n!},$$

$$= {}_{q+1}F_q\left(\frac{\alpha}{k},\frac{\beta_1}{k},\dots,\frac{\beta_q}{k};\frac{\gamma_1}{k},\dots,\frac{\gamma_q}{k};\frac{px}{s}\right), \tag{5.1}$$

for $\alpha \in C;\ \gamma_j \in C\backslash z_0^-, j = 1,2\dots,q,;\ k,p,s \in \mathbb{R}^+$.

Remark.5.1. If we set $s = p = k = 1$ in the equation (5.1), then we get the generalized

hypergeometric function [15]

$${}_{q+1}F_q^{1,1,1}(\alpha,\beta_1,\dots,\beta_q;\gamma_1,\dots,\gamma_q;x) = {}_{q+1}F_q(\alpha,\beta_1,\dots,\beta_q;\gamma_1,\dots,\gamma_q;x)$$
$$= \sum_{n=0}^{\infty}\frac{(\alpha)_n(\beta_1)_n\dots(\beta_q)_n}{(\gamma_1)_n\dots(\gamma_q)_n}\frac{x^n}{n!}. \tag{5.2}$$

Remark. 5.2. If we set $q = 1$in equation (5.1), then we get the three parameter Gauss hypergeometric function

$${}_2F_1^{k,p,s}(\alpha,\beta;\gamma;x)$$
$$= \sum_{n=0}^{\infty}\frac{(\alpha)_{n,k}^{p,s}(\beta)_{n,k}^{p,s}}{(\gamma)_{n,k}^{p,s}}\frac{x^n}{n!} = {}_2F_1\left(\frac{\alpha}{k},\frac{\beta}{k};\frac{\gamma}{k};\frac{px}{s}\right). \tag{5.3}$$

Theorem 5.1. If $Re(\gamma_j) > Re(\beta_j) > 0\,; j = 1,2\dots,q;$ $\gamma_j \in C\backslash z_0^-\,;\ k,p,s \in \mathbb{R}^+$, then

$${}_{q+1}F_q^{k,p,s}(\alpha,\beta_1,\dots,\beta_q;\gamma_1,\dots,\gamma_q;x)$$
$$= \frac{1}{\prod_{j=1}^q {}_pB_k^s(\beta_j,\gamma_j-\beta_j)}$$
$$\times\left(\frac{1}{k}\right)^q\int_0^1\dots\int_0^1\prod_{j=1}^q\left[t_j^{\frac{\beta_j}{k}-1}(1-t_j)^{\frac{\gamma_j-\beta_j}{k}-1}\right]$$
$$\times\left(1-\frac{t_1\dots t_j px}{s}\right)^{\frac{-\alpha}{k}}dt_1\dots dt_q\,. \tag{5.4}$$

Proof. Employing (4.3) in (5.1), we have

$${}_{q+1}F_q^{k,p,s}(\alpha,\beta_1,\dots,\beta_q;\gamma_1,\dots,\gamma_q;x)$$
$$= \sum_{n=0}^{\infty}(\alpha)_{n,k}^{p,s}\frac{{}_pB_k^s(\beta_1+nk,\gamma_1-\beta_1)}{{}_pB_k^s(\beta_1,\gamma_1-\beta_1)}\dots\frac{{}_pB_k^s(\beta_q+nk,\gamma_q-\beta_q)}{{}_pB_k^s(\beta_q,\gamma_q-\beta_q)}\frac{x^n}{n!}.$$

Employing (4.2) and the binomial theorem we obtain the desired result.

Corollary 5.1. Taking $q = 1$ in (5.4), we get

$${}_2F_1^{k,p,s}(\alpha,\beta;\gamma;x) = \frac{1}{k\,{}_pB_k^s(\beta,\gamma-\beta)}$$
$$\times\int_0^1 t^{\frac{\beta}{k}-1}(1-t)^{\frac{\gamma-\beta}{k}-1}\left(1-\frac{pxt}{s}\right)^{\frac{-\alpha}{k}}dt. \tag{5.5}$$

Corollary 5.2.

$${}_2F_1^{k,p,s}\left(\alpha,\beta;\gamma;\frac{s}{p}\right) = \frac{{}_p\Gamma_k^s(\gamma)\,{}_p\Gamma_k^s(\gamma-\alpha-\beta)}{{}_p\Gamma_k^s(\gamma-\alpha)\,{}_p\Gamma_k^s(\gamma-\beta)}\,. \tag{5.6}$$

Proof. Setting $x = \frac{s}{p}$ in (5.5), we obtain

$$ {}_2F_1^{k,p,s}\left(\alpha,\beta;\gamma;\frac{s}{p}\right) = \frac{1}{k\ {}_pB_k^s(\beta,\gamma-\beta)}\int_0^1 t^{\frac{\beta}{k}-1}(1-t)^{\frac{\gamma-\alpha-\beta}{k}-1}\,dt, = \frac{{}_pB_k^s(\beta,\gamma-\alpha-\beta)}{{}_pB_k^s(\beta,\gamma-\beta)}. $$

which by using (4.1), we get the desired result.

Further, if we put $\alpha=-nk$ in (5.6) and using (3.4), then we obtain

$$ {}_2F_1^{k,p,s}\left(-nk,\beta;\gamma;\frac{s}{p}\right) = \frac{(\gamma-\beta)_{n,k}^{p,s}}{(\gamma)_{n,k}^{p,s}}. \qquad (5.7) $$

Theorem 5.2. If $\alpha_j \in C;\ \beta_j \in C\backslash z_0^-; j=1,2\dots,q$; $Re(\gamma)>0$, then

$$ \frac{{}_p\Gamma_k^s(\beta_q+\gamma)}{{}_p\Gamma_k^s(\beta_q)\ {}_p\Gamma_k^s(\gamma)}\int_0^1 t^{\frac{\beta_q}{k}-1}(1-t)^{\frac{\gamma}{k}-1} \times {}_{q+1}F_q^{k,p,s}\left(\alpha_{1,}\dots,\alpha_{q+1};\beta_1,\dots,\beta_q;xt\right)dt = k\,{}_{q+1}F_q^{k,p,s}\left(\alpha_1,\dots,\alpha_{q+1};\beta_1,\dots,\beta_q+\gamma;x\right). \qquad (5.8) $$

Proof. Denoting the left-hand side of (5.8) by I, using (5.1) and (3.4), we get

$$ I=\sum_{n=0}^{\infty}\frac{(\alpha_1)_{n,k}^{p,s}\dots(\alpha_{q+1})_{n,k}^{p,s}}{(\beta_1)_{n,k}^{p,s}\dots(\beta_{q-1})_{n,k}^{p,s}}\frac{x^n}{n!}\frac{{}_p\Gamma_k^s(\beta_q+\gamma)}{{}_p\Gamma_k^s(\gamma)\ {}_p\Gamma_k^s(\beta_q+nk)} \times\int_0^1 t^{\frac{\beta_q+nk}{k}-1}(1-t)^{\frac{\gamma}{k}-1}dt, $$

Using (4.2), we get

I =

$$ \sum_{n=0}^{\infty}\frac{(\alpha_1)_{n,k}^{p,s}\dots(\alpha_{q+1})_{n,k}^{p,s}}{(\beta_1)_{n,k}^{p,s}\dots(\beta_{q-1})_{n,k}^{p,s}}\frac{x^n}{n!}\frac{k\ {}_p\Gamma_k^s(\beta_q+\gamma)}{{}_p\Gamma_k\ \ (\gamma)\ {}_p\Gamma_k^s(\beta_q+nk)} \times {}_pB_k^s(\beta_q+nk,\gamma). $$

Finally, employing (4.1) and (3.4), we get the desired result.

Corollary 5.3. Setting $q=1,\beta_1=\beta$ in (5.8), we have

$$ \frac{{}_p\Gamma_k^s(\beta+\gamma)}{{}_p\Gamma_k^s(\beta)\ {}_p\Gamma_k^s(\gamma)} \times\int_0^1 t^{\frac{\beta}{k}-1}(1-t)^{\frac{\gamma}{k}-1}\ {}_2F_1^{k,p,s}(\alpha_1,\alpha_2;\beta;xt)dt = k\ {}_2F_1^{k,p,s}(\alpha_1,\alpha_2;\beta+\gamma;x)\,. \qquad (5.9) $$

Corollary 5.4. Setting $q=2,\ ,\beta_1=\beta,\ \beta_2=\alpha_3$ and replacing γ by $\gamma-\alpha_3$ in (5.8), we have

$$ \frac{{}_p\Gamma_k^s(\gamma)}{{}_p\Gamma_k^s(\alpha_3)\ {}_p\Gamma_k^s(\gamma-\alpha_3)}\int_0^1 t^{\frac{\alpha_3}{k}-1}(1-t)^{\frac{\gamma-\alpha_3}{k}-1} \times\ {}_2F_1^{k,p,s}(\alpha_1,\alpha_2;\beta;xt)dt = k\ {}_3F_2^{k,p,s}(\alpha_1,\alpha_2,\alpha_3;\beta,\gamma;x). \qquad (5.10) $$

Theorem 5.3. For $Re(\gamma-\alpha-\beta)>0;\ \gamma\in C\backslash z_0^-, k,p$ $,s\in\mathbb{R}^+$, we have

$$ {}_3F_2^{k,p,s}\left(-mk,\alpha,\beta;(1-m)k-\gamma+\alpha+\beta,\gamma;\frac{s}{p}\right) = \frac{(\gamma-\alpha)_{m,k}^{p,s}(\gamma-\beta)_{m,k}^{p,s}}{(\gamma)_{m,k}^{p,s}(\gamma-\alpha-\beta)_{m,k}^{p,s}}. \qquad (5.11) $$

Proof. The Euler's transformation of the Gaussian hypergeometric function is given as follows (see [15]):

$$ {}_2F_1\ \ (\alpha,\beta;\gamma\,;x) = (1-x)^{\gamma-\alpha-\beta}\ {}_2F_1\ \ (\gamma-\alpha,\gamma-\beta;\gamma\,;x). \qquad (5.12) $$

Using (5.12) in the right-hand side of (5.3), we get

$$ {}_2F_1^{k,p,s}(\alpha,\beta;\gamma;x) = \left(1-\frac{xp}{s}\right)^{\frac{\gamma-\alpha-\beta}{k}}\ {}_2F_1^{k,p,s}(\gamma-\alpha,\gamma-\beta;\gamma;x), \qquad (5.13) $$

or

$$ {}_2F_1^{k,p,s}(\gamma-\alpha,\gamma-\beta;\gamma;x) = \left(1-\frac{xp}{s}\right)^{-\left(\frac{\gamma-\alpha-\beta}{k}\right)}\ {}_2F_1^{k,p,s}(\alpha,\beta;\gamma;x), $$

which can be written as

$$ \sum_{m=0}^{\infty}\frac{(\gamma-\alpha)_{m,k}^{p,s}(\gamma-\beta)_{m,k}^{p,s}}{(\gamma)_{m,k}^{p,s}}\frac{x^m}{m!} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(\gamma-\alpha-\beta)_{m,k}^{p,s}\,x^m}{m!}\frac{(\alpha)_{n,k}^{p,s}(\beta)_{n,k}^{p,s}}{(\gamma)_{n,k}^{p,s}}\frac{x^n}{n!}. $$

Employing the following identities:

$$ \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}A(n,m) = \sum_{m=0}^{\infty}\sum_{n=0}^{m}A(n,m-n), $$

$$ (m-n)! = \frac{(-1)^n\,m!}{(-m)_n}, $$

we get

$$ \sum_{m=0}^{\infty}\frac{(\gamma-\alpha)_{m,k}^{p,s}(\gamma-\beta)_{m,k}^{p,s}}{(\gamma)_{m,k}^{p,s}}\frac{x^m}{m!} = $$

$$ \sum_{m=0}^{\infty}\sum_{n=0}^{m}\frac{(\gamma-\alpha-\beta)_{m-n,k}^{p,s}(-m)_n\,x^{m-n}}{(-1)^n\,m!}\frac{(\alpha)_{n,k}^{p,s}(\beta)_{n,k}^{p,s}}{(\gamma)_{n,k}^{p,s}}\frac{x^n}{n!}. $$

Employing the identities (2.2) and (2.5), we get

$$\sum_{m=0}^{\infty}\frac{(\gamma-\alpha)_{m,k}^{p,s}(\gamma-\beta)_{m,k}^{p,s}}{(\gamma)_{m,k}^{p,s}}\frac{x^m}{m!}$$

$$=\sum_{m=0}^{\infty}\frac{(\gamma-\alpha-\beta)_{m,k}^{p,s}x^m}{m!}$$

$$\times\ {}_3F_2^{k,p,s}\left(-mk,\alpha,\beta;(1-m)k-\gamma+\alpha+\beta,\gamma;\frac{s}{p}\right).$$

Finally, comparing the coefficient of x^m on both sides of the above equation we get the desired result.

Theorem 5.4. For $Re(\gamma)>Re(\beta)>0, k,p,s\in\mathbb{R}^+$, $q\in Z^+, |x|<1$, the following integral representation of the three-parameter generalized hypergeometric function ${}_{q+1}F_q^{k,p,s}$ hold true:

$${}_{q+1}F_q^{k,p,s}\left(\alpha,\frac{\beta}{q},\frac{\beta+k}{q},\dots,\frac{\beta+(q-1)k}{q};\frac{\gamma}{q},\frac{\gamma+k}{q},\dots,\frac{\gamma+(q-1)k}{q};x\right)$$

$$=\frac{1}{k\ {}_pB_k^s(\beta,\gamma-\beta)}\int_0^1 t^{\frac{\beta}{k}-1}(1-t)^{\frac{\gamma-\beta}{k}-1}\left(1-\frac{pxt^q}{s}\right)^{\frac{-\alpha}{k}}dt. \tag{5.14}$$

Proof. The integral representation of ${}_{q+1}F_q$ is given by [18]

$${}_{q+1}F_q\left(\alpha,\frac{\beta}{q},\frac{\beta}{q}+\frac{1}{q},\dots,\frac{\beta}{q}+\frac{q-1}{q};\frac{\gamma}{q},\frac{\gamma}{q}+\frac{1}{q},\dots,\frac{\gamma}{q}+\frac{q-1}{q};x\right)$$

$$=\frac{1}{B(\beta,\gamma-\beta)}\int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-xt^q)^{-\alpha}\,dt. \tag{5.15}$$

Also from (5.1), we have

$${}_{q+1}F_q^{k,p,s}\left(\alpha,\frac{\beta}{q},\frac{\beta+k}{q},\dots,\frac{\beta+(q-1)k}{q};\frac{\gamma}{q},\frac{\gamma+k}{q},\dots,\frac{\gamma+(q-1)k}{q};x\right)$$

$$={}_{q+1}F_q\left(\frac{\alpha}{k},\frac{\beta/k}{q},\frac{\beta/k}{q}+\frac{1}{q},\dots,\frac{\beta/k}{q}+\frac{q-1}{q};\frac{\gamma/k}{q},\frac{\gamma/k}{q}+\frac{1}{q},\dots,\frac{\gamma/k}{q}+\frac{q-1}{q};\frac{px}{s}\right). \tag{5.16}$$

Using (5.15) in the right-hand side of (5.16), we have

$${}_{q+1}F_q^{k,p,s}\left(\alpha,\frac{\beta}{q},\frac{\beta+k}{q},\dots,\frac{\beta+(q-1)k}{q};\frac{\gamma}{q},\frac{\gamma+k}{q},\dots,\frac{\gamma+(q-1)k}{q};x\right)$$

$$=\frac{1}{B\left(\frac{\beta}{k},\frac{\gamma-\beta}{k}\right)}$$

$$\times\int_0^1 t^{\frac{\beta}{k}-1}(1-t)^{\frac{\gamma-\beta}{k}-1}\left(1-\frac{pxt^q}{s}\right)^{\frac{-\alpha}{k}}dt. \tag{5.17}$$

Finally, using (4.2), we have the desired result.

Corollary 5.5. Setting $q=2$ in (5.14), we get

$${}_3F_2^{k,p,s}\left(\alpha,\frac{\beta}{2},\frac{\beta+k}{2};\frac{\gamma}{2},\frac{\gamma+k}{2};x\right)$$

$$=\frac{1}{k\ {}_pB_k^s(\beta,\gamma-\beta)}$$

$$\times\int_0^1 t^{\frac{\beta}{k}-1}(1-t)^{\frac{\gamma-\beta}{k}-1}\left(1-\frac{pxt^2}{s}\right)^{\frac{-\alpha}{k}}dt. \tag{5.18}$$

Corollary 5.6. Setting $q=3$ in (5.14), we get

$${}_4F_3^{k,p,s}\left(\alpha,\frac{\beta}{3},\frac{\beta+k}{3},\frac{\beta+2k}{3};\frac{\gamma}{3},\frac{\gamma+k}{3},\frac{\gamma+2k}{3};x\right)$$

$$=\frac{1}{k\ {}_pB_k^s(\beta,\gamma-\beta)}\int_0^1 t^{\frac{\beta}{k}-1}(1-t)^{\frac{\gamma-\beta}{k}-1}\left(1-\frac{pxt^3}{s}\right)^{\frac{-\alpha}{k}}dt. \tag{5.19}$$

Remark 5.3. If we put $s=1, p=k$ in the results (5.14), (5.18) and (5.19), then we get certain a known integral representation of k- generalized hypergeometric functions ([19],[20]).

Theorem 5.5. If $Re(\gamma-\alpha-\beta)>0, \gamma\in C\backslash z_0^-$, then

$${}_3F_2^{k,p,s}\left(\alpha,\frac{\beta}{2},\frac{\beta+k}{2};\frac{\gamma}{2},\frac{\gamma+k}{2};\frac{s}{p}\right)$$

$$=\frac{{}_p\Gamma_k^s(\gamma)\ {}_p\Gamma_k^s(\gamma-\alpha-\beta)}{{}_p\Gamma_k^s(\gamma-\alpha)\ {}_p\Gamma_k^s(\gamma-\beta)}\ {}_2F_1^{k,p,s}\left(\alpha,\beta;\gamma-\alpha;-\frac{s}{p}\right). \tag{5.20}$$

Proof. Setting $x=\frac{s}{p}$ in (5.18), we obtain

$${}_3F_2^{k,p,s}\left(\alpha,\frac{\beta}{2},\frac{\beta+k}{2};\frac{\gamma}{2},\frac{\gamma+k}{2};\frac{s}{p}\right)$$

$$=\frac{{}_p\Gamma_k^s(\gamma)}{k\ {}_p\Gamma_k^s(\beta)\ {}_p\Gamma_k^s(\gamma-\beta)}$$

$$\times\int_0^1 t^{\frac{\beta}{k}-1}(1-t)^{\frac{\gamma-\beta-\alpha}{k}-1}(1+t)^{\frac{-\alpha}{k}}dt.$$

Using binomial theorem and the results (4.1) and (4.2) in the above equation, we have

$${}_3F_2^{k,p,s}\left(\alpha,\frac{\beta}{2},\frac{\beta+k}{2};\frac{\gamma}{2},\frac{\gamma+k}{2};x\right)$$

$$=\frac{{}_p\Gamma_k^s(\gamma)\ {}_p\Gamma_k^s(\gamma-\alpha-\beta)}{{}_p\Gamma_k^s(\gamma-\alpha)\ {}_p\Gamma_k^s(\gamma-\beta)}$$

$$\times\sum_{n=0}^{\infty}\left(\frac{\alpha}{k}\right)_n\frac{{}_p\Gamma_k^s(\beta+nk)\ {}_p\Gamma_k^s(\gamma-\alpha)}{{}_p\Gamma_k^s(\beta)\ {}_p\Gamma_k^s(\gamma-\alpha+nk)}\frac{(-1)^n}{n!}.$$

Finally, employing the identities (2.2) and (3.4), we have the desired result.

Corollary 5.7. Setting $\alpha=-nk$ in (5.20) and using (3.4), we obtain

$${}_3F_2^{k,p,s}\left(-nk,\frac{\beta}{2},\frac{\beta+k}{2};\frac{\gamma}{2},\frac{\gamma+k}{2};\frac{s}{p}\right)$$

$$=\frac{(\gamma-\beta)_{n,k}^{p,s}}{(\gamma)_{n,k}^{p,s}}\ {}_2F_1^{k,p,s}\left(-nk,\beta;\gamma+nk;-\frac{s}{p}\right). \tag{5.21}$$

Theorem 5.6. If $Re(\gamma-\alpha-\beta)>0, \gamma \in C\backslash z_0^-$, then

$$ {}_4F_3^{k,p,s}\left(\alpha,\frac{\beta}{3},\frac{\beta+k}{3},\frac{\beta+2k}{3};\frac{\gamma}{3},\frac{\gamma+k}{3},\frac{\gamma+2k}{3};\frac{s}{p}\right) $$

$$ =\frac{{}_p\Gamma_k^s(\gamma)\,{}_p\Gamma_k^s(\gamma-\alpha-\beta)}{{}_p\Gamma_k^s(\gamma-\alpha)\,{}_p\Gamma_k^s(\gamma-\beta)}\sum_{n=0}^{\infty}\left(-\frac{s}{p}\right)^n\frac{(\alpha)_{n,k}^{p,s}(\beta)_{n,k}^{p,s}}{(\gamma-\alpha)_{n,k}^{p,s}n!} $$

$$ \times\ {}_2F_1^{k,p,s}\left(-nk,\beta+nk;\gamma-\alpha+nk;-\frac{s}{p}\right). \qquad (5.22) $$

Proof. Setting $x=\frac{s}{p}$ in (5.19), we obtain

$$ {}_4F_3^{k,p,s}\left(\alpha,\frac{\beta}{3},\frac{\beta+k}{3},\frac{\beta+2k}{3};\frac{\gamma}{3},\frac{\gamma+k}{3},\frac{\gamma+2k}{3};\frac{s}{p}\right) $$

$$ =\frac{1}{k\ {}_pB_k^s(\beta,\gamma-\beta)}\int_0^1 t^{\frac{\beta}{k}-1}(1-t)^{\frac{\gamma-\beta-\alpha}{k}-1}\left(1+t(1+t)\right)^{\frac{-\alpha}{k}}dt. $$

Using binomial theorem and simplify by using the results (4.1) and (4.2) in the above equation, we have

$$ {}_4F_3^{k,p,s}\left(\alpha,\frac{\beta}{3},\frac{\beta+k}{3},\frac{\beta+2k}{3};\frac{\gamma}{3},\frac{\gamma+k}{3},\frac{\gamma+2k}{3};\frac{s}{p}\right) $$

$$ =\frac{{}_p\Gamma_k^s(\gamma)\,{}_p\Gamma_k^s(\gamma-\alpha-\beta)}{{}_p\Gamma_k^s(\gamma-\alpha)\,{}_p\Gamma_k^s(\gamma-\beta)} $$

$$ \times\sum_{n=0}^{\infty}\sum_{r=0}^{n}\left(\frac{\alpha}{k}\right)_n\frac{{}_p\Gamma_k^s(\beta+nk+rk)\,{}_p\Gamma_k^s(\gamma-\alpha)(-n)_r}{{}_p\Gamma_k^s(\beta)\,{}_p\Gamma_k^s(\gamma-\alpha+nk+rk)}\frac{(-1)^{n+r}}{n!\,r!} $$

Employing the identities (2.2) and (3.4), we have

$$ {}_4F_3^{k,p,s}\left(\alpha,\frac{\beta}{3},\frac{\beta+k}{3},\frac{\beta+2k}{3};\frac{\gamma}{3},\frac{\gamma+k}{3},\frac{\gamma+2k}{3};\frac{s}{p}\right) $$

$$ =\frac{{}_p\Gamma_k^s(\gamma)\,{}_p\Gamma_k^s(\gamma-\alpha-\beta)}{{}_p\Gamma_k^s(\gamma-\alpha)\,{}_p\Gamma_k^s(\gamma-\beta)} $$

$$ \times\sum_{n=0}^{\infty}\sum_{r=0}^{n}\left(\frac{s}{p}\right)^n\left(\frac{s}{p}\right)^r\frac{(\alpha)_{n,k}^{p,s}(\beta)_{n+r,k}^{p,s}(-nk)_{r,k}^{p,s}}{(\gamma-\alpha)_{n+r,k}^{p,s}}\frac{(-1)^{n+r}}{n!\,r!}. $$

Finally using (2.4), we have the desired result.

6. Conclusion

In this work, we have introduced the *(k-p-s)* - Pochhammer's symbol, *(k-p-s)*- Gamma function, *(k-p-s)*- Beta function and *(k-p-s)*- generalized hypergeometric function. Many properties of these extended functions were studied, including their integral representations, summation formulas and functional relations. Each of the results derived in this work are reduce either to a known or new identities for certain generalized functions given in the literature. For example, if $s=1, p=k$, then the results are reduce to the results of k-functions (see [14],[19],[20]). If $s=1$, then we obtain the results of *p*-*k*-functions [16]. If $k=\frac{p}{s}$, then we obtain the results involving *p*-*s*-functions [17]. Further, if we put $s=p=k=1$, then we get the classical results (see for example [18]). Future work may focus on applications of the proposed functions in fractional calculus, integral transforms and solutions of generalized differential equations

References

[1] E. Ozergin, M. A. Ozarslan and A. Altin, "Extension of gamma, beta and hypergeometric functions", *Journal of Computational and Applied Mathematics*, vol. 235, pp. 4601– 4610, 2011.

[2] A. Al-Gonah and W. K. Mohammed, "A New Extension of extended gamma and beta functions and their properties", *Journal of Scientific and Engineering Research*, vol. 5, no.9, pp. 257-270, 2018.

[3] A.A. Atash, S.S. Barahmah and M.A. Kulib, "On a new extension of extended gamma and beta function", *International Journal of Statistics and Applied Mathematics*, vol. 3, no. 6, pp. 14-18, 2018.

[4] M. Shadab, S. Jabee and J. Choi, "An extension of beta function and its Application", *Far East Journal of Mathematical Sciences*, vol. 103, no. 1, pp. 235-251, 2018.

[5] M.A. Kulib, F.B.F. Mohsen and S.S. Barahmah, "Further extended gamma and beta functions in terms of generalized Wright function", *Electronic Journal of university of Aden for Basic and Applied Sciences*, vol.1, no. 2, pp. 77-83, 2020.

[6] K. S. Gehlot and K. S. Nisar, "Extension of two parameter gamma, beta functions and its properties", *Applications and Applied Mathematics*, vol. 15, no. 6, pp. 39-55, 2020.

[7] M. K. Rasheed and A.H. Majeed, "Generalization of gamma and beta functions with certain properties and statistical application", *Iraqi Journal of Science*, vol. 64, no. 7, pp. 4487- 4495, 2023. DOI: 10.24996/ijs.2023.64.7.36.

[8] Cetinkaya, M. B. Yabasan and I. O. Kymaz, "The extended Srivastava's triple hypergeometric functions and their integral representations", *Journal of Nonlinear Sciences and Applications*, vol. 10, pp. 1–7, 2016.

[9] M. Shadab and J. Choi, "Extensions of Appell and Lauricella's hypergeometric Functions", *Far East Journal of Mathematical Sciences 102*, pp. 1301–1317, 2017.

[10] A.A. Atash, S.S. Barahmah and M.A. Kulib, "On Extensions of extended Gauss hypergeometric function", *Communications in Advanced Mathematical Sciences*, vol. 2, no. 3, pp.199-205, 2019.

[11] A.A. Atash and M.A. Kulib, "Extension of Extons hypergeometric function K16", *CUBO, A Mathematical Journal*, vol. 23, no. 3, pp. 489–501, 2021.

[12] S. Halici and A. Cetinkaya, " k-Srivastava hypergeometric functions and their integral representations", *Miskolc Mathematical Notes*, vol. 25, no. 2, pp. 685–697, 2024. DOI: 10.18514/MMN.2024.4351

[13] H. S. Bellehaj and A. A. Atash, "The extended Exton's quadruple hypergeometric function K (α, β)15, p and its properties", *J. Innov. Appl. Math. Comput. Sci*, vol. 5, no. 1, pp. 53–65, 2025. DOI: 10.58205/jiamcs. v5i1.1916

[14] R. Diaz and E. Pariguan, "On hypergeometric functions and Pochhammer k-symbol", *Divulg. Mat*. vol.15, no. 2, pp. 179-192, 2007.

[15] H. M. Srivastava and J. Choi, "Zeta and q-Zeta Functions and Associated Series and Integrals", *Elsevier Science Publishers, Amsterdam, London and New York*, 2012.

[16] K. S. Gehlot, "Two parameter gamma function and its properties", *arXiv preprint*, arXiv:1701.01052v1 [math. Ca], 2017.

[17] S. Mubeen, S.A.H. Shah, G. Rahman, K. S. Nisar and T. Abdeljawad, "Some generalized special functions and their properties", *Advances in the Theory of Nonlinear Analysis and its Applications 1*, no. 1, pp. 45-65, 2022. Available: https://doi.org/10.31197/atnaa.768532.

[18] K. A. Driver and S. J. Johnston, "An integral representation of some three parameter hypergeometric functions", *Electronic Transactions on Numerical Analysis*, vol. 25, pp. 115-120, 2006.

[19] Ali and M. Z. Iqbal, "Integral representation of some basic k-hypergeometric Functions", *J. Appl. Math. & Informatics*, vol. 40, no. 1 - 2, pp. 205 – 213, 2022. Available: https://doi.org/10.14317/jami.2022.205.

[20] S. Mubeen and G.M. Habibullah, "An integral representation of some k-hypergeometric functions", *Int. Math. Forum. 7*, pp. 203-207, 2012.

مقالة بحثية

حول تعميمات (*K,P,S*) لدوال غاما وبيتا والدوال فوق الهندسية وخصائصها

ميسون أحمد كليب[1,*]، و أحمد علي عتش[2]

[1] قسم العلوم الأساسية (الرياضيات)، كلية الهندسة، جامعة عدن، عدن، اليمن
[2] قسم الرياضيات، كلية التربية، جامعة شبوة، شبوة، اليمن

*** الباحث الممثّل: ميسون أحمد كليب؛ البريد الالكتروني: maisoonahmedkulib@gmail.com**

استلم في: 26 يناير 2026 / قبل في: 09 فبراير 2026 / نشر في 31 مارس 2026

المُلخّص

في هذه الورقة، نقدم تعميمًا موحدًا بثلاثة معاملات ((*k,p,s*)) لرمز بوشهامر (Pochhammer) ودالتي غاما وبيتا. وبالاستناد إلى هذه التعريفات، يتم تعريف دالة فوق هندسية معممة من النوع ((*k,p,s*)). يتم اشتقاق عدة خصائص أساسية، بما في ذلك المعادلات الدالية، وصيغ الجمع، والتمثيلات التكاملية. ويُظهر أنه يمكن الحصول على العديد من التوسعات المعروفة للدوال الخاصة كحالات خاصة ضمن الإطار المقترح، مما يؤدي إلى توحيد وتوسيع النتائج السابقة في الأدبيات العلمية.

الكلمات المفتاحية: رمز بوشهامر-(k-p-s)؛ دالة غاما-(k-p-s)؛ دوال بيتا-(k-p-s)؛ الدالة فوق الهندسية المعممة-(k p s)؛ التمثيلات التكاملية.

تصنيف MSC 2010: B1533، C0533، C2033

How to cite this article:

M. A. Kulaib, and A. A. Atash, "ON (*K, P, S*)-GENERALIZATIONS OF THE GAMMA, BETA AND HYPERGEOMETRIC FUNCTIONS AND THEIR PROPERTIES", *Electron. J. Univ. Aden Basic Appl. Sci.*, vol. 7, no. 1, pp. 9-17, Mar. 2026. DOI: https://doi.org/10.47372/ejua-ba.2026.1.494